The Arithmetic Teacher

FEBRUARY • 1959

THE ARITHMETIC TEACHER

a journal of The National Council of Teachers of Mathematics

Editor: BEN A. SUELTZ, State University Teachers College, Cortland, N. Y.
Associate Editors: MARGUERITE BRYDEGAARD, San Diego State College, San Diego, Calif.; E. GLENADINE GIBB, State Teachers College, Cedar Falls, Iowa; JOHN R. CLARK, New Hope, Pa.; JOSEPH J. URBANCEK, Chicago Teachers College, Chicago 21, Ill.

All editorial correspondence, including books for review, should be addressed to the Editor. Advertising correspondence, subscriptions to THE ARITHMETIC TEACHER, and notice of change of address should be sent to:

The National Council of Teachers of Mathematics
1201 Sixteenth St., N.W., Washington 6, D. C.

Entered as second-class matter at the post office at Washington, D. C., June 21, 1954, under act of March 3, 1879. Additional entry granted at Menasha, Wis.

THE ARITHMETIC TEACHER is published six times per year, in October, November, December, February, March and April. The individual subscription price of \$5.00 (\$1.50 to students) includes membership in the Council (For an additional \$3.00 (\$1.00 to students) the subscriber may also receive THE MATHEMATICS TEACHER.) Institutional Subscription: \$7.00 per year. Single copies: 85¢ each. Remittance should be made payable to the National Council of Teachers of Mathematics, 1201 Sixteenth St., N. W., Washington 6, D. C. Add 25¢ per year for mailing to Canada, 50¢ per year for mailing to other foreign countries.

Table of Contents

THE ARITHMETIC TEACHER

Volume VI Number 1

February 1959

Mathematics Education in the Soviet 7-Year School*

(for pupils age 7 to 14 years)

A Report prepared by the Methodological Section on Teaching Mathematics in the Secondary School of the Research Institute of Methods of Teaching under the direction of NIKOLAI F. CHETVERUKHIN, *Presidium Member*.

General Remarks

THE SOVIET INTERMEDIATE SCHOOL ("Mittelschule") for the development of general knowledge is included in a total school period of 10 years. The first four schoolyears, which are attended by pupils beginning with their 7th year of age, are called the elementary school ("Elementarschule"). At this school one teacher gives the lessons in all subjects. The children learn reading and writing as well as the fundamental operations of arithmetic and they become familiar with the most simple phenomena of nature and society.

Beginning with the 5th school year, all teaching is carried on by specialized teachers and classes are conducted only by those who have studied either at state universities or pedagogical institutes.†

In 1943 general compulsory school attendance for a period of seven years was introduced in USSR, first in industrial capitals and later all over the country. The three instructional levels, elementary (1–4), intermediate (5–7) and secondary (8–10) form the Soviet Unity School for the development of general knowledge. Mathematics lessons are given in all ten years of study in the unity school ("Einheitsschule")—6 lessons per week. In the course of ten years 1980 lessons in mathematics are given. This is 20 per cent of the total number of lessons for all subjects throughout the 10 school years.

The following table shows the distribution of weekly hours in the several subjects of mathematics for the first seven years. After that there is given the syllabus for the 7-year school.*

Classes:	I	II	III	IV	V	VI	VII
Arithmetic	6	6	6	6	6	2	—
Algebra	—	—	—	—	—	2	4
Geometry	—	—	—	—	—	2	2
Trigonometry	—	—	—	—	—	—	—
Total	6	6	6	6	6	6	6

* This condensed report was translated from the original German Text by Howard F. Fehr.

† Pedagogical Institutes are comparable to our Teacher Training Colleges preparing secondary school teachers in major subject areas.

* The Russian schoolyear consists of 33 full weeks (no holidays) 6 days each week or 198 school days per year. The class period, beginning with the 5th class (year) is 45 minutes.

Subjects Treated in Mathematics at the Soviet 7-Year School

The syllabus of this school comprises mathematical instruction of a practical and general nature, the mastery of which is necessary for a pupil's promotion to the 8th class.

Teaching of mathematics starts with arithmetic. In this subject pupils learn the rules and fundamental operations of arithmetic with whole and fractional numbers, get acquainted with the propositions and law of arithmetic as well as with the basic principles of solutions of problems.

In classes I to IV pupils are taught elementary arithmetic. In particular in these classes through oral and written work they learn the cardinal and ordinal numbers and the four fundamental operations with natural numbers. They become familiar with simple properties of the most important plane figures and solids and the calculation of area and volume. In the 5th and 6th classes this general knowledge is broadened and deepened, and subsequent stress is laid on common and decimal fractions.

In the algebra lessons in the 6th and 7th classes the nature of number is profoundly explained by the introduction of negative numbers. Great store is placed in identical transformation and equations. Simple functions and their graphical representation are introduced, and pupils learn to solve problems with the aid of equations.

Apart from the foregoing, pupils of the 6th and 7th classes are also given lessons in geometry. In this subject they are taught the properties of simple geometrical plane figures.

The Soviet intermediate school ("Mittelschule") has been in existence for a long time, and during the course of years, methodical guiding principles for teachers of mathematics have been elaborated. In these directions the subject matter to be treated is spelled out precisely and the teaching methods to be used by all teachers in all mathematical subjects is described. The syllabus and the annotations to the syllabus are decisive in directing teachers to the knowledge to be conveyed to the pupils and the abilities to be developed. Extracurricular activities, such as study groups, in addition to lessons at school are widely in vogue at Soviet schools. Pupils who are particularly interested in mathematics are trained in these study groups.

The syllabus for mathematics is as follows:

Arithmetic

The aim of mathematical instruction is 1) to teach the pupils to carry out knowingly, rationally and speedily the four fundamental operations with whole and fractional numbers in writing and orally, and 2) to use the pupils' knowledge of these operations for the solution of practical problems and questions.

Correct procedure for mental arithmetic is of great importance for the development of general knowledge: Whereas in the case of written exercises the operations with whole and fractional numbers are carried out according to fixed rules, mental arithmetic offers many possibilities of developing the pupils' initiative and creative powers.

Mental arithmetic also has great methodical influence, for it facilitates written calculations, guarantees their correctness and saves much time during the lesson at school as well as in carrying out the pupil's home work.

According to the syllabus the use of the Russian arithmetical chart is strongly recommended for instructional purposes. This chart is employed all over Russia as a very practical teaching aid. The introduction into the use of this instrument is regarded as a means of rationalizing theoretical operations of arithmetic and a lead to applying arithmetic to practical life. In each stage of learning not only principles, theorems and rules are taught but the solution of problems is also explained.

Arithmetical problems* are of great im-

* *Translator's note:* This refers to problems involving the structure and rationale of arithmetic and not to so called word problems. It is really elementary number theory.

portance as they contribute to the demonstration and acquisition of the main principles of arithmetic and enable the pupils to realize functional interrelations between different factors. The solution of arithmetical problems furthers the development of logical thinking and linguistic expression. Lessons in arithmetic have not only the aim of teaching the routine methods of solving problems, but also to develop the pupils' logical thinking, their initiative and creative powers.

The syllabus recommends the solving of practical problems which are related to problems of socialist development, geometrical situations, and those which particularly lay stress on a combination of theory and practice. The subject matter of arithmetic is distributed as follows:

1st to the 4th class

(6 lessons per week in each class)

1. Arithmetical operations with whole numbers.

5th class

(6 lessons per week; 198 per year)

1. Recapitulation and systematization of the subject matter taught in the lower 4 classes—20 lessons (8 hours)*
2. Divisibility of numbers—20 lessons (8 hours).
3. Common fractions—90 lessons (36 hours).
4. Decimal fractions—50 lessons (20 hours).
5. Practical problems—6 lessons (———)
6. Recapitulation—12 lessons (6 hours).

6th Class

(2 lessons per week; 66 per year)

1. Percentage calculus—20 lessons (10 hours)
2. Proportions; direct and indirect proportions—32 lessons (16 hours)
3. Recapitulation—14 lessons (7 hours).

Algebra

The aims of lessons in algebra are as follows; in classes 6–7: a) extension of the class of numbers dealt with in arithmetic (rational numbers, approximate computation and representation on straight line segments; and other considerations of the results of measurements by finite decimal fractions; b) development of practice in the application of algebraic symbols; c) explanation of special methods (tricks) of identical transformation and the solution of algebraic equations and equation systems of the first degree, developing at the same time the children's facility through practice in the fundamental operations; d) development of the pupils' knowledge of the methods of solution and ability to apply this knowledge to the solution of practical problems.

In the 6th and 7th class the group of number shall be confined to whole numbers, common fractions and finite decimal fractions. During this first stage of algebraic instruction, however, strict attention is to be paid to the fact that pupils round off numbers with an appropriate degree of accuracy and that they calculate the value of a square root with the aid of logarithm tables as accurately as permissible. The same holds true for the transformation of common fractions into decimal fractions. All this plays an important part in mathematical instruction, as the pupils must be in a position to apply the practical fundamental operations to the solution to problems in the field of geometry, physics and technics. With respect to "identical transformation," e.g. the resolution of equations and systems of equations, the following recommendations are made in the syllabus: a) pupils shall learn only such transformations which are necessary for further studies of algebra or those which are applicable to technical science; b) selection of exercises which are of greatest use for acquiring comprehensive mathematical knowledge; c) the solution of algebraic equations, i.e. equations with letters instead of numbers (in the 6th and 7th class) follows 2

* The first figure indicates the approximate number of lessons at school whereas the figure in parentheses shows the approximate number of hours provided for the pupils' homework.

aims, to improve the pupils' calculation technique and to ascertain the variability of an expression and with graphic representation to numerical values, in particular, graphic representations and the use of x-axis and y-axis serve this purpose well (5th and 6th class). By means of drawings on ruled paper pupils become familiar with examples of simple functional relations (direct, indirect and linear relations) and learn how to describe these relations in a coordinate system (7th class).

The knowledge of geometrical principles will certainly help the pupils to understand fully the nature of algebra. The elaboration of drawings of geometrical figures at class or at home increases the pupils' interest in this subject, develops their activity and teaches them all the special methods (tricks) of this subject. The syllabus for algebra lessons is as follows:

6th Class

(2 lessons weekly—66 per year)

1. Algebraic formulas and equations—15 lessons (7 hours).
2. Positive and negative numbers—20 lessons (10 hours).
3. Operation with integral algebraic expressions—31 lessons (15 hours).

7th Class

(4 lessons per week; 132 per year)

1. Resolution into factors—36 lessons (18 hours).
2. Algebraic fractions—24 lessons (12 hours).
3. Equations of the first degree with 1 unknown quantity—34 lessons (17 hours).
4. Equations with 2 unknown quantities; systems of equations—28 lessons (14 hours).
5. Recapitulation—10 lessons (5 hours).

Geometry

Systematic study of geometry begins in the 6th class. Lessons in arithmetic in classes 1–5 teach the pupils some fundamental knowledge of geometry. This knowledge principally concerns measuring of straight lines, plane areas, and volumes and has the object of laying the foundation for further geometrical studies. This basic knowledge of arithmetic is also necessary for the study of physics. Lessons in geometry are of special importance, because a) pupils acquire logical and systematical knowledge, become familiar with the methods of geometry and develop impressions of space, b) the syllabus for lessons in geometry is arranged in such a manner as to prepare for further study of this subject, c) systematic learning of geometry according to the present syllabus trains the pupils in logical thinking, and d) geometric teaching promotes the development of the pupils' practical abilities. The most important task of the school today is to convey polytechnical education. In order to fulfill this task the present syllabus of geometry provides for more practical applications in each class and specifies all the exercises to be carried out in each respective class. The geometrical subject matter is distributed according to the syllabus as follows:

6th Class

(2 lessons per week; 66 per year)

1. Basic principles—12 lessons (6 hours).
2. Parallel straight lines—22 lessons (11 hours).
3. Triangle—28 lessons (15 hours).
4. Practical instruction, land survey—4 lessons.

7th Class

2 lessons per week; 66 per year)

1. Quadrangle—26 lessons (13 hours).
2. Circle—34 lessons (17 hours).
3. Practical instruction, land survey—6 lessons (17 hours).

The mathematics syllabus for classes 1–7 includes only such subjects that are accessible and able to be mastered by pupils of the respective age and the importance of which has been proved by long-years' experience in pre-revolutionary and Soviet schools. The new work set for these classes is closely related to polytechnical education

and requires the solution of a series of problems with the following objectives: 1) Lessons in arithmetic are to increase the ability in written and mental arithmetic, in particular the use of approximate calculation, 2) algebra lessons have the special purpose of developing the pupils' knowledge of functional relations, 3) lessons in geometry are to be given in such a form as to introduce gradually the methods of deduction. In the beginning, experimental teaching and induction are to play a more important role. Lastly, in all subjects, mathematics shall give preference to problems involving measuring, land survey, modeling and the solution of problems chosen from practical life.

EDITOR'S NOTE. We are indebted to Professor Fehr for obtaining and translating this important report. The Russians take their school work very seriously with six days per week in classes and with homework. It is apparent that their program is more mathematical than ours and that the attention to principles and propositions soon goes beyond the levels we expect with comparable age groups. It is also interesting to note the use of study groups for auxiliary learning for the more able pupils whereas in this country we have tended to use extra time to aid the weak and indolent. The written work is by fixed rule whereas mental work aims to develop initiative. This report as well as others mentions the use of applications of mathematical principles and processes and hence the study is not confined to abstract and theoretical propositions.

What are the implications for this country? Should our public schools operate six days per week? Should they continue through the summer? Should we have more direct teaching of rules and procedures with a higher value placed upon memory and duplication of things learned? Should we expect more definite learnings at each grade level and higher-level learnings with greater depth of understanding? Should we introduce more advanced topics at earlier grade levels? Can we discard some of the topics we are now teaching? Can we expect that a child at the end of grade eight might well have covered the material in our present grade nine and even some of grade ten? Should we seek to change a general attitude toward school and education? Does possession of a high school diploma imply all it might and perhaps should imply? Ought our schools be more standardized in curricula and in achievement? One can ask many many questions about our schools but it is more difficult to effect change even when change is indicated and agreed upon.

DEVICE REVIEW

Arithme-Stick. Ginn and Company, 1958. $4.50.

This device for manipulative study of number consists of five heavy plastic sticks and enough markers which slip on tracks to mount up to twenty markers per stick. The sticks fit nicely together forming either a horizontal number frame or a vertical abacus and separate for use as single number lines used either horizontally or vertically.

The material seems adapted to the purpose; the colors, black and orange, make the arrangements easy to see and the workmanship seems good. Everything worked as described on the model examined. The durability should be satisfactory.

Eight different uses of the Arithme-Sticks are suggested and illustrated in the sixteen page manual together with suggestions for simple games and examples of their use to illustrate certain simple computations. One side allows the collection of groups of ten behind a stop while the unused markers are hidden. The other side of each Arithme-Stick provides marks to guide the student in the use of the markers to record parts of a group.

This is a simple and usable device which has the good features of several single devices. It can be arranged, for instance, as either a 9-bead or a 10-bead abacus and it allows the number lines or other arrangements to be viewed either horizontally or vertically. It provides for almost automatic recognition of and emphasis upon groups of ten.

It is not complicated to handle and in the hands of children it will not lend itself readily to any particular bias or system which the teacher or the textbook may have but should be an aid to discovery by the learner at many stages in his development.

E. W. HAMILTON

Beginnings of Mathematical Education in Russia

JOHN DE FRANCIS
Quinnipiac College, Hamden, Conn.

ALL THINGS ARE SAID to have a beginning and an end; some certainly have more than one of each. Sputnik is one end of the Soviet educational system. I should like to focus attention on one small but vital beginning, that of the first formal introduction of mathematical concepts at the start of the educational process.

My study begins with a close examination of a Russian textbook entitled *Arithmetic. Textbook for the First Grade of Primary School.** As the title page states, the book was "approved" by the Russian Ministry of Education, and its backing by the Russian Academy of Pedagogical Sciences is also indicated. In short, the work is not an isolated private venture, as are textbooks in the United States, but is an officially sponsored undertaking.

Contents-wise there are several outstanding differences between this work and comparable American arithmetics. Some of these will be noted toward the end of the article. First, however, I should like to provide the reader with a basis for making his own comparison of points which particularly interest him. To this end I shall devote a good portion of the article to a rather detailed summary of the work.

The first part of the book, comprising about two-fifths of the 142 pages, is entitled "First Ten." It begins with a single page devoted to the concept of relative size. The words "larger-smaller" and "longer-shorter" are presented, and the concepts are illustrated with colored drawings of balls and pencils. (Illustrations are used profusely on the first 27 pages, sparingly thereafter. Several colors are used here also, but only black and white in the rest of the book.)

The next two pages stress rational counting and one-to-one correspondence. After being asked "how many" objects (up to ten) are depicted, the pupil is further told to put down "as many sticks" or "as many [paper] squares" as there are objects.

A single page follows on relative position. The pupil is asked whether objects are above or below, on the left or the right.

Pages 7–29 are devoted to presenting the numbers 1 through 10.

An illustration of a single boy and a group of people presents the concepts, also given in word form, of "many-one." Several single objects are depicted, including one bead on a wire, one dot in a small rectangle, and a one-kopek coin. The numeral is presented, and the pupil is told to write it under his drawing of one mushroom.

In introducing the number 2, stress is placed on its "twoness." A bicycle, a pair of shoes, and a pair of skis are depicted. How many legs does a rooster have? a hen? How many ears on a rabbit and wings on a bird? Further, the 2-kopek coin, 2 beads on a string, and 2 dots are shown. The pupil is asked to put down one little stick, then another, and tell how many there are. He is also asked to arrange sticks in the pattern of a V and an inverted V.

For the numbers 3 and 4 there is similar emphasis on "threeness" and "fourness," and on arranging sticks in the form of a triangle (not designated as such, however) and a square (in this case so designated). Square counters are arranged in a particular pattern, and the pupil is asked how else it is possible to arrange the same number of squares. Each of the numbers is presented as one more than the preceding. The pupil is asked to draw a stated number of objects and to write the appropriate numeral underneath.

* A. S. Pchyolko and G. B. Polyak, *Arithmetika. Uchebnik dlya pervogo klassa nachalnoi shkoly.* Moskva, 1955.

With the number 5, besides exercises such as those for numbers 1 to 4, statements in the form $1+1=2$ and $4+1=5$ are introduced, and the pupil is asked to solve problems such as $4+1=$ __.

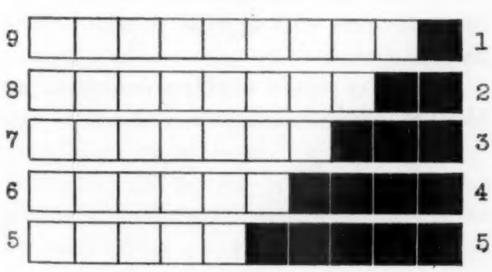
An additional exercise involving 6 requires the pupil to state how it is possible to arrange 6 little cakes on 2 saucers. Another calls for identifying which numbers are missing in $\boxed{1}\ \square\ \square\ \boxed{4}\ \square\ \boxed{6}$. The pupil is asked to count from 1 to 6 and from 6 to 1. For the first time the text takes up subtraction. Initially no word problems involving subtraction are presented. The pupil is first introduced to such forms as $2-1=1$ and $6-1=5$ and is also asked to solve problems such as $5-1=$ __.

Under the number 7 a word problem involving subtraction of 1 from 7 is given, followed immediately by $7-1=6$.

Presentation of 8 utilizes no new type of exercise.

Under 9 the word "triangle" is used in connection with the request to arrange sticks according to the illustration given (three triangles).

The only new feature in the presentation of 10 is the following illustration:



The section devoted to introducing numbers 1 to 10 is followed by 24 pages of exercises in addition and subtraction. "Bridging" is not involved as yet. Subsections take up in turn adding and subtracting 1, adding and subtracting 2, and so on. Zero is taught as "a case of subtraction, when the remainder is 0." The notation that $2-2=0$ is followed immediately by exercises such as $3-3$, $8-8$, $6-3-3$. Exercises such as the following are presented:

$1+1$	$5+1+1$	$8-8$
$9+1$	$7+2+1$	$3+7-19$
$9-1$	$10-3-5$	$9-6-\ 3$

Stress is placed on pairs like

$$9+1 \qquad\qquad 10-1$$

The following type of exercise occurs frequently:

$$\frac{7+3=}{7+1+1+1} \quad \frac{6+4=}{6+2+2} \quad \frac{7-2=}{7-1-1} \quad \frac{9-4=}{9-2-2}$$

Word problems abound. Here are a few:

"There are 7 days in a week. Children do not attend school one day in the week. How many school days are there in a week?"

"Natasha first drew 6 little fir trees, and then another 4. How many fir trees did she draw in all?" "Natasha drew 10 little fir trees. She colored 2 fir trees. How many fir trees remained for her to color?"

"From what numbers is it possible to take 3, [4, 5, etc.]?"

"Put down 3 sticks on the left and as many on the right. How many sticks are there in all?"

A distinctive feature of this section, and indeed of the rest of the book itself, is the heavy emphasis on the pupil making up problems of his own. For example: "Make up a problem in which it is necessary to add 4 to 5." "On a shelf are 7 plates. Sima took down 4 plates. Make up a question and solve the problem." "Make up four examples of subtraction of 4 and solve them."

The addition facts studied so far are now brought together in a table which can be summarized as follows:

$1+1=2$	$1+2=3$	$1+3=4$	$1+9=10$
$\vdots$	$\vdots$	$\vdots$	
$9+1=10$	$8+2=10$	$7+3=10$	

Attention is turned briefly to measurement. The pupil is asked to measure the length of the classroom in paces and in meters. Also: "Measure off a thread 5 meters long. Cut off two meters from it. What is the

length of the remaining thread?" "A carpenter sawed off 3 meters from a board, and afterward another 2 meters. Make up a question and solve the problem."

The section on the First ten ends with a brief review. The requirement that the pupil make up his own word problems is pushed a step further in the following two problems: "Vasya bought a book and an album. The album was 6 rubles, and the book ______ rubles. How much were the book and the album together? Complete and solve the problem." "A father bought his son a little pail and a shovel. How much money did he pay for this purchase? Complete and solve the problem."

The next part of the book (pp. 59–126) is entitled "Second Ten." It begins by depicting ten as a single bundle of ten sticks and as a continuous lower layer of 10 blocks, topped by from one to 10 other blocks to represent the numbers from 11 to 20. The pupil is asked how many tens and ones there are in fifteen, eighteen, nineteen, and to name the numbers made up to 1 ten and 5 ones, 1 ten and 9 ones, 2 tens. These questions are also used to teach place-value with the following table:

Tens	Ones

He is given problems involving addition, first without bridging,* then with bridging. Subtraction is presented first without borrowing, then with borrowing. Here are various examples of the two processes:

11+ 8	3+3+10	18− 3−2
5+13	14+3+2	19−10−4
9+ 3	15−3	16− 9+6
5+ 9	16−9	16−12+7

* The term "bridging" means using a basic combination such as 9+3 as a base and extending the learning into higher decades such as 19+3, 29+3, etc.

Another way in which subtraction is presented is as follows:

$$\frac{16-2=}{6-2=\ 4}$$
$$10+4=14$$

The pupil is asked to count by two, by four, and by five to 20, and by three and by six to 18. (Here for the first time the words to "six," not the numerals, are used.) He is also asked to "increase 13 by 5," "increase 1 by 19," "decrease 14 by 4." A good deal of emphasis is placed on word problems involving numerical comparisons, such as "The width of one room is 6 meters, and another room is 3 meters wider. What is the width of the other room?" (The more literal translation "wider by 3 meters" relates this exercise more closely to the previously mentioned "decrease 14 by 4.") One of the forms taken by this sort of problem is the question "What number is 2 less than 14? 1 less than 15? 4 less than 18?"

Following this come "two-step problems," some of which involve measurements in kilograms and liters. Example: "A collective farmer poured 12 liters of milk into one can, and 4 less than this into another can. How many liters of milk did she pour into both cans?"

By way of review at this point, problems like the following are presented:

8+ . . . =12
3+ . . . =20
. . . + 1 =11
. . . + 5 =14
. . . + 15 =17

Attention is directed toward multiplication and division. The former is introduced with the request to count by 2 to 20. Multiplication is compared with addition:

2+2=	2×2=
2+2+2+2+2=	2×5=
2+2+2+2+2+2+2+2	2×8=

Similarly the pupil is asked to count by 3, 4, 5, 6. The multiples are depicted as a pair of acorns for 2, three circles arranged as a triangle for 3, four circles arranged in a square for 4, five carrots in a bunch for 5. Many word problems appear: "Vera bought 8 toys at 2 rubles each to decorate a fir tree and gave the clerk 20 rubles. How much change should she get?" "A collective farmer milked some cows and poured 8 liters of milk into each of two cans, and 4 liters into a third can. How many liters of milk did she get?" "Set up a problem like the preceding which would be solved thus:

1. $9L \times 2 = 18L$
2. $18L + 2L = 20L$"

The multiplication table is given for products not exceeding 20.

Division, further described as division into so-and-so many parts, is contrasted with multiplication:

1×2	5×4
$2 \div 2$	$20 \div 4$

Division is drilled by use of pure number problems like $8 \div 2$, $18 \div 6$ and also in both one-step and two-step word problems, including the following: "In one week 13 kilograms of oats were fed to some geese, and in the second week one kilogram more than this. How many kilograms of oats were used a day in the second week?"

Finally, the section on the Second Ten concludes with the following summary of the four processes:

$3 \times 4 - 9$	$2 \times 8 \div 4$	$9 \times 2 - 13$
$16 \div 2 + 5$	$15 \div 3 \times 2$	$20 \div 4 + 14$
$7 \times 2 - 8$	$9 \div 9 \times 8$	$3 \times 4 - 12$
$18 \div 3 - 14$	$4 \times 5 \div 10$	$18 \div 9 + 9$

The final section, entitled "The First Hundred," is introduced by an illustration of two groups of five bundles each containing ten sticks. After a question on how many sticks in one bundle and how many sticks in all, the pupil is asked to put down "4 tens of sticks," "9 tens of sticks," and so on. Then he is asked to put down "forty sticks," "ninety sticks," etc., and to state how many tens there are in each of these numbers. Another exercise requires putting down 8 tens of sticks, then 5 sticks, and stating how many sticks there are in all. Place-value is indicated as follows:

Tens	Ones

The pupil is asked to place the "numeral 3" in the left column and the "numeral 5" in the right column, and to read the number obtained. He also reads a string of numbers: 27, 43, 78. . . . A rule is given: "Ones are written in the first place on the right, tens in the second." Then the pupil is told, "Write with numerals the numbers thirty-five, sixty-eight, forty-six, ninety-two." A table is given of the numbers 1–100, and the pupil is asked to find various numbers in it.

Measurement is extended to include centimeters: The fact that "There are 100 centimeters in one meter" is also presented as follows:

$$1\,\text{m} = 100\ \text{cm}$$

Practice is given in addition and subtraction of tens:

$$2\ \text{tens} + 1\ \text{ten} = 3\ \text{tens}$$
$$20 + 10 = 30$$

$$3\ \text{tens} - 1\ \text{ten} = 2\ \text{tens}$$
$$30 - 10 = 20$$

$$10 + \cdot\ \cdot\ \cdot = 30$$
$$\cdot\ \cdot\ \cdot + 20 = 30$$

Also: "Increase 40 by 20; 80 by 20; 70 by 30." "Decrease 50 by 10; 70 by 30; 80 by 20." In word problems the pupil is taught the distinction between one-step and two-step problems by being asked to make up some of his own:

"In a factory settlement are two kindergartens, 80 children in one and 20 less in

the other. Make up a question such that the problem can be solved in one step."

"In a pioneer camp there are two detachments: 40 persons in one and 10 less in the other. Make up a question such that the problem can be solved in two steps."

Multiplication and division of tens are given much the same treatment:

$$2 \text{ tens} \times 3 = 6 \text{ tens}$$
$$20 \times 3 = 60$$

$$4 \text{ tens} \div 2 = 2 \text{ tens}$$
$$40 \div 2 = 20$$

A mixture of all four processes includes the following:

$20 \times 4 - 50$	$2 \times 9 \div 6$	$14 \div 7 \times 8$
$30 \times 3 + 10$	$7 + 8 - 6$	$50 \times 2 \div 5$
$60 \div 3 \times 4$	$16 - 9 + 5$	$80 \div 4 \div 2$

The following is an example of the word problems which conclude this final section:

"Pioneers planted 90 flower seedlings—10 of them in a round flower bed, and the rest equally in four triangular beds. How many seedlings did they plant in each triangular bed?"

Comparison with American Schools

In passing now to the comparison of typical American and Russian textbooks of arithmetic, considerable difficulty is encounted in determing just what to compare. While first grade in Russian schools is more or less equivalent to first grade in American schools, the pupils are not exactly comparable. Russian children start first grade at age seven, American youngsters at age six. There might be some point, therefore, in comparing the arithmetic of Russian 7 year-olds with the arithmetic of American 7 year-olds, i.e. of American second graders. I shall therefore compare first grade arithmetic in Russia with that of both the first and the second grades of this country.

In general the topics covered in the Russian work are far in excess of those of our first-grade texts. The content exceeds that covered in our second grade books in many points and falls short in a few. Even in those cases in which the same topics are covered there is considerable difference in emphasis. This is particularly true of what might be called the preliminaries to number work, such as the concepts of one-to-one and one-to-many correspondence, rote and rational counting, and so on. The elaborate treatment which we accord to these topics is lacking in the Russian text.

The text makes no attempt to contrast the cardinal and ordinal principles, and indeed minimizes the latter. The words for "first" and "second" appear incidentally on page 127, and the word for "third" on page 109. All three are also used in section headings (pp. 3, 59, 126). I have not noted other ordinals, either in full form or in the form Number 1, Number 2, etc. In contrast, some American texts, notably the Riess-Hartung *Developing Number Readiness** and related materials, devote much attention to the ordinal concept and to its presentation before the cardinal concept.

The word form of numbers also receives little attention. "One" occurs on page 7; special words for "two," "three," "four," and "five" appear subsequently in haphazard fashion. At page 127, in connection with the presentation of numbers from 21 to 100, several number words like "thirty-five" are used in such a way that the pupil is assumed to be able to read all the numbers in word form. Our first and second grade arithmetic books, while more limited in the number of such words used, are more systematic in their presentation.

Grouping is given some attention in the Russian text but has not been clearly thought through. Some use is made of model groups for the smaller numbers and of structural groups for larger numbers. Yet not nearly so much is presented along these lines as the copious colored illustrations of our books, much less the specially prepared ma-

* New York: Scott, Foresman and Company, 1946.

terials of Catherine Stern's "Structural Arithmetic." The pupil himself is expected to do a good deal in the way of working out structural patterns of numbers. Utilizing sticks, squares, and disks, he imitates geometric figures like triangles and squares and experiments with the ways in which the elements making up numbers can be grouped.

One point in which our texts go a bit further than the Russian is in the introduction of fractions. The Russian work makes no mention whatsoever of the subject. Some American texts present the fraction $\frac{1}{2}$ in the first grade and $\frac{1}{3}$ and $\frac{1}{4}$ in the second.

The subject of time is also ignored in the Russian text. Measurement is fairly extensively practiced only in respect to length and weight, and even this presents less of a problem than our weights and measures since Russian children, happily for them, need contend only with the relatively simpler metric system.

A striking fact about the Russian test is its failure to present a single example of vertical addition or subtraction. American arithmetics, on the other hand, present these processes chiefly in vertical form, though a good deal of attention is given, at least in the second grade, to the horizontal presentation as well.

Another point of difference between the two arithmetics is in the size of vocabulary used. American textbook writers, working perhaps under the influence of prevailing reading theory on limitation of vocabulary, consciously restrict the number of words used in the arithmetic books. In one series,* for example, the total vocabulary of the first grade test is limited to 40 words, and to 22 additional words in the second grade. In checking the Russian arithmetic on this point I found over 100 words in the first dozen pages, which come well before the introduction of word problems. With these the vocabulary soars. My impression (I gave up on the actual counting) is that little if any restriction was exercised on the number of words used, which probably exceeds a thousand and may number several thousands.

The number and complexity of the word problems in the Russian text also present a marked difference from American arithmetics. The text contains 892 numbered exercises. Some three hundred of these consist of one to two dozen number problems each (e.g. $10-9$, $14+2-5$, $20\times4-50$). Most of the remainder are word problems examples of which were given earlier.

The tremendous amount of material presented is not matched by any marked superiority in psychological insights. To give only one example, in introducing the number 8 the text has an illustration of a pod containing 8 peas, a useless illustration since the 8 peas cannot be viewed as 8 but can be identified as such only by counting. In this respect the book is closer to mediocre American texts than to more insightful treatments by some of our exponents of meaningful arithmetic.

Although there are other differences which might be noted it would seem well to stop at this point and dispose briefly of the inevitable question as to what all this means in terms of comparative achievement. If so inclined one might assert that our first graders lag far behind Russian first-graders and that our textbooks should therefore be expanded to include all the material covered by the Russians. Or one might advance the diametrically opposite view that our first graders, who at six years of age start school a year ahead of their Russian counterparts, need an extra year of maturation and therefore should be given no arithmetic whatsoever. Such divergent views underline the dangers in reaching over-simplified conclusions. Much more than what has been discussed here needs to be taken into consideration. One would like to know, for example, what the exact answer is to the question of how much that is in the text actually gets into the heads of Russian first

(*Continued on page 16*)

* Dale Carpenter and Mae Knight Clark, *Find Out About Numbers* (New York: Macmillan, 1957). Dale Carpenter, Mae Knight Clark, and Esther J. Swenson, *More About Numbers* (New York: Macmillan, 1957).

Bibliography of Books for Enrichment in Arithmetic

Adrien L. Hess
Montana State College, Bozeman

Writers and illustrators are devoting much time and talent to producing books which can be used for enrichment and for appreciation units in arithmetic, for the upper grades in particular. While many of the books for the lower grades are essentially counting books, an occasional one goes beyond this and introduces arithmetic addition in a clever manner.

The present bibliography supplements and brings up to date the bibliography by Hutcheson, Mantor and Holmberg[1] and the bibliography by Hess.[2]

[1] Hutcheson, Ruth; Mantor, Edna and Holmberg, Marjorie. The Elementary School Mathematics Library—A Selected Bibliography, The Arithmetic Teacher. Feb. 1956. 3: 8–16.

[2] Hess, Adrien L. A Bibliography of Mathematics Books for Elementary School Libraries—The Arithmetic Teacher. Feb. 1957. 4: 15–20.

Lower Grades

	Grade	Price
Ambler, C. Gifford. TEN LITTLE FOXHOUNDS. Illustrated. 23 pp. 1958. Children's Press	1, 2, 3	$2.30
A book with excellent illustrations in color. The verse describes counting out of ten foxhounds from the first one having a rabbit race to the last lonely little foxhound going home to hide leaving the "old fox laughing 'til he cried."		
Elkin, Benjamin. SIX FOOLISH FISHERMEN. Illustrated by Katherine Evans. 32 pp. 1957. Children's Press	K, 1, 2, 3	2.50
An old folk tale, illustrated in full color, of six brothers who went fishing. At the end of a day's fishing each counted only five brothers. Where was the sixth? A small boy helped them count to find the sixth brother. He was given all the fish and everyone went home happy.		
Fraser, Phyllis. COUNTING RHYMES. Illustrated by Corrine Malvern. 22 pp. 1947. Simon and Schuster	K, 1, 2	1.12
An excellent collection of counting rhymes illustrated in colors. Some of the rhymes are old favorites such as Going to St. Ives, Monday's Child and Thirty Days Hath September. Two rhymes are devoted to Addition and Subtraction. The selection is quite wide.		
Higgins, Loyta. LET'S SAVE MONEY. Illustrated by Violet LaMont. 18 pp. 1958. Simon and Schuster	3, 4	.25
From the many pictures of coins in the book youngsters learn about pennies, nickels, dimes, and quarters. The book provides a movable wheel in the front cover which holds more than a dollar's worth of coins. The book encourages the saving of money.		

	Grade	Price
JACKSON, KATHRYN. SCHOOL DAYS. Pictures by Violet LaMont. 64 pp. 1954. Simon and Schuster. A Golden Book of Easy-to-Read Stories and Things to Do.	K, 1, 2	1.00
The book is devoted largely to the alphabet and reading. Nine pages in color and verse are devoted to numbers. Everyday articles such as pets, houses, trees, etc., in color, are used to teach counting from 1 through 8. The numbers are written in symbols and in words.		
LANGSTAFF, JOHN. OVER IN THE MEADOW. Illustrated by Feodor Rojankovsky. 32 pp. 1957. Harcourt.	K, 1, 2, 3	2.75
An old counting song for children with a simple new melody and amusing verses is presented in color and line. Wild animals in natural habitat constitute the objects from one to ten which are counted. Its effectiveness would be enhanced if sung by children as well as adults.		
LAWELL, BETTY LOU. TOOOOOT! A TRAIN WHISTLE COUNTING BOOK. Illustrated by Paul Julian. 26 pp. 1958 Melmont. .	K, 1, 2, 3	
The whistle of a train attracts the attention of everyone. Any boy interested in trains can imagine himself the engineer and may count the number of toots from one to five of the whistle for each occasion. Young and old can sing "The Train Whistle Song" at the end of the book.		
MERWIN, DECIE. TIME FOR TAMMIE. Author illustrated 40 pp. 1946. Walck.	K, 1, 2	1.75
Tammie, almost six, made a game out of telling time by pretending that her arms were the hands of a clock. Then she was able to teach the game to an older boy who had befriended her. Young readers will want to play the clock game like Tammie.		
MOORE, LILLIAN. COUNT TO TEN. Illustrated by Beth Krush. 20 pp. 1957. Simon and Schuster.	K, 1, 2	1.12
The illustrations in several colors of children playing with familiar objects such as one red ball, three wheels on a tricycle, five children in a sandbox, etc., will attract and hold the attention of all youngsters. The numerals from one to ten appear on each page, as well as objects representing a particular number. A well-bound book.		
MOORE, LILLIAN. MY BIG GOLDEN COUNTING BOOK. Illustrated by Garth Williams. 32 pp. 1956. Simon and Schuster. . . .	1, 2, 3	1.69
Extra large pages allow ample space for beautiful pictures of one dog, two lambs, etc., up to ten acorns. All pictures and numbers are merged into two pages near the end of the book. The last pages show the symbol and word for each number as well as the objects depicting the number. Format seems quite large.		
PETER, JOHN. WHAT TIME IS IT? Illustrated by Joseph Zabinski. 20 pp. 1954. Wonder Books.	1, 2, 3	.25
The significant hours of a child's day are represented by a series of clock faces. These are accompanied by illustrations in color of familiar activities that usually take place at that time of day. On the back cover is the face of a clock for the youngster to practice telling time by drawing the hands of the clock with crayon.		

	Grade	Price
PETER, JOHN. THE COUNTING BOOK. Illustrated by Bob Riley. 20 pp. 1957. Wonder Books	1, 2, 3	.25
This is more than a counting book. Directions are given for cutting each page of the book into three parts. Each of these parts has one or more objects in color to illustrate the number found on that part. The top and middle parts give the numbers to be added. The correct answers are on the third parts. The youngster must look for the correct answer to the particular combination of objects presented. The sums are from 2 to 10 with more emphasis on sums of 6, 7, 8, 9, 10.		
SEIGNOBOSC, FRANCOISE. JEANNE MARIE COUNTS HER SHEEP. Author illustrated. 32 pp. 1951. Scribner	K, 1, 2, 3	2.00
An informal introduction to numbers in verse and easy to count colored pictures is made as Jeanne Marie considers the things she wishes—if Patapon, her sheep, has a lamb, or two, or three—or more.		
SLOVODKIN, LOUIS. ONE IS GOOD. Author illustrated. 25 pp. 1956. Vanguard	K or younger	2.50
The idea—one is good, but two is better—is well illustrated to attract youngsters. The climax is reached when there are two or three or more than four and then all can sing and play. This book has a rather limited use for counting, but was not meant for a counting book. The idea of playing together was inspired by the author-illustrator's twin grandsons.		
WATSON, NANCY DINGMAN. ANNIE'S SPENDING SPREE. Illustrated by Aldren Watson. 45 pp. 1957. Viking	K, 1, 2, 3	2.50
When Annie was given a dollar bill the day before her birthday, she learned about the parts of a dollar. Her adventures in visiting stores to spend the dollar further stressed this and at the same time many other number relationships are introduced. Pictures in four colors.		
WITHERS, CARL. COUNTING OUT. Illustrated by Elizabeth Ripley. 46 pp. 1946. Walck	3, 4, 5	.75
Ninety-six verses that can be used to count out. The humorous illustrations make this a good book for older children to read and chuckle over. An excellent book for a teacher to use for new twists to old games.		
———. 1, 2, 3. Illustrated by Art Seiden 20 pp. 1958. Grosset	K or younger	1.50
The numerals from one to ten appear in color of page size, accompanied by a picture of a baby, two dogs, five rabbits, etc., also in color. The size of the numerals and illustrations makes the book most usable with small children. Heavy pages and easy-to-clean covers make this book excellent for family use with a tiny tot who is just beginning to recognize objects.		

Upper Grades

	Grade	Price
ADLER, IRVING. MAGIC HOUSE OF NUMBERS. Illustrated by Ruth Adler. 128 pp. 1957. Day	6 up	2.95

This is more than a collection of curiosities, riddles, tricks, and games. It gives an introduction to our number system as well as bases other than ten and shows hidden steps and the "how" and "why" of many operations. Readers are encouraged to devise new tricks and games. Answers are given to problems and puzzles.

LEE, RECTOR LAWRENCE. GIL'S DISCOVERY IN THE MINE. Illustrated by Sidney A. Quinn. 202 pp. 1957. Little, Brown and Co.	7 up	3.00

In this novel, Gil, who is interested only in football, finds that low grades in Plane Geometry kept him off the team part of his junior year. During a summer's work in a mine, Gil is adroitly maneuvered into surveying, bookkeeping and real problems in engineering. This brings him to see the importance of mathematics and to develop a liking for it.

MEYER, JEROME. FUN WITH MATHEMATICS. Illustrated. 176 pp. 1952. World	7 up	2.75

The book is filled with a wide assortment of mathematical tricks, things to do and interesting problems written in a popular style. A number of topics are quite advanced, but any reader can find many that will intrigue. Some will want to make a slide rule from an ordinary rule. Almost ninety figures help to explain the book.

PLOTZ, HELEN. IMAGINATION'S OTHER PLACE. Poems of Science and Mathematics. Illustrated with wood engravings by Clare Leighton. XIII+200 pp. 1955. Crowell	7 up	3.50

This collection of poems emphasizes that mathematics, science and poetry have much in common. Among the 26 poems in the section "Kingdom of Numbers," one finds space-time and quartz crystal clocks, the icosasphere and Euclid. One concludes that mathematics is productive in humor as well as in seriousness.

RAVIELLI, ANTHONY. AN ADVENTURE IN GEOMETRY. Author illustrated. 117 pp. 1957. Viking	7 up	3.00

Many vivid drawings of geometric forms and pictures of objects in nature which portray these forms should intrigue young and old alike. The concepts illustrated range from simple ones like points, lines and angles to more complicated ones like conic sections, cycloid, spirals and projective geometry. The author's concise style is most effective.

SAXON, G. R. HOW FAST? Illustrated by Isabel Sherman Harris. 32 pp. 1954. Crowell	5, 6	2.00

In this age of rockets, any boy will want to compare the speed of animals and machines. The black and white illustrations are just as lively as the animals and machines portrayed. An interest arousing book, HOW FAST? should prove to be easy reading for intermediate grades and contains facts that many older boys and even adults enjoy discussing.

	Grade	Price
TANNEBAUM, BEULAH AND STILLMAN, MYRA. UNDERSTANDING MAPS. Charting the Land, Sea and Sky. Illustrated by Rus Anderson. 144 pp. 1957. McGraw-Hill......................	7 up	3.00
The many pictures, charts and diagrams make this a source book for any youngster interested in maps of the world about him. Under different mappings—such as Mercator, Projector or Cylindrical Projection—the distortions of the earth are sure to intrigue the reader. The "Now Try This" encourages the reader to see that a given technique works.		
ZARCHY, HARRY. WHEEL OF TIME. Illustrated by Rene Martin. 144 pp. 1957. Crowell..................................	7 up	2.75
The history of the development of man's concept of time, the instruments he invented to measure it, and the calendars showing year, month, week, day, and season is interestingly discussed. The relationship of time and navigation is stressed and attention is directed to latitude, longitude and the time zones. The drawings and illustrations add much to the book.		

Directory of Publishers

CHILDREN'S PRESS. Children's Press, Jackson Blvd. and Racine Ave., Chicago 7, Illinois.

CROWELL. The Thomas Y. Crowell Company, 432 Fourth Avenue, New York 16, Yew York.

DAY. John Day Company, Inc., 62 West 45th Street, New York.

GROSSET. Grosset and Dunlap, 1107 Broadway, New York 10, New York.

HARCOURT. Harcourt, Brace and Company, Inc., 383 Madison Avenue, New York 17, New York.

LITTLE. Little, Brown and Company, Boston, Massachusetts.

MELMONT. Melmont Publishers, Inc. (A Division of Carl J. Leibel), 1236 South Hatcher Avenue, La Puente, California.

SCRIBNER. Charles Scribner's Sons, 597 Fifth Avenue, New York 17, New York.

SIMON AND SCHUSTER. Simon and Schuster, Inc., 630 Fifth Avenue, New York 20, New York.

VANGUARD. Vanguard Press, 424 Madison Avenue, New York 17, New York.

VIKING. The Viking Press, Inc., 625 Madison Avenue, New York 22, New York.

WALCK. Henry Z. Walck, Inc., 101 Fifth Avenue, New York 3, New York.

WHITTLESEY. Whittlesey House, 330 West 42nd Street, New York 36, New York.

WORLD. World Book Company. 2231 West 110th Street, Boston 8, Massachusetts.

YOUNG SCOTT BOOKS. Young Scott Books, 8 West 13th Street, New York 11, New York.

Beginnings of Mathematical Education in Russia

(Continued from page 11)

graders. Only by taking into account this and many other complex aspects of the problem at hand, a task beyond the scope of the present article, could one attempt any fully satisfactory evaluation and comparison.

EDITOR'S NOTE. Mr. De Francis has made a very good analysis of the content of a mathematics book for seven-year-old pupils in Russia and then has made some comparisons with texts for that age group in this country. One notes a good deal of similarity and is also impressed with the more rapid progression in Russia. Much of the value in a primary education depends upon how the learning is conducted and on that topic we have limited information. It does appear that thinking and not rote memorization are features. The pupil is expected to work out his own structural patterns of number using sticks etc. Apparently thinking and discovery are considered important. One is also impressed by the large number of "word problems" and with the apparent disregard for difficulty of vocabulary. Perhaps we have over-emphasized vocabulary difficulty in this country. If a word is needed, why not teach it? Are we allowing our education to be dominated by our concern with the median and perhaps even the 25th percentile?

Using Teachers' Manuals for Deeper Learning

EVELYN W. FOOTE
Montclair Public School, New Jersey

WHAT CAN BE DONE in the classroom to heighten experiences in arithmetic, to foster sound reasoning, to produce good thinkers, to promote sensible approaches to problem-solving, to develop skills in computation? Much depends upon the interests, desires, and competencies of the teacher, plus her understanding of the processes concomitant to learning and her patience in dealing with the learners themselves.

There are many helps for teachers on the present-day market. Some are in the form of manuals, more popularly known as teachers' editions, that accompany the various textbooks; some are prepared and distributed by educational associations such as the N.E.A. or A.C.E.I.; others appear as service bulletins issued by publishers of school books or are found in professional magazines like "The Grade Teacher," "The Instructor," and "American Childhood." Several companies are manufacturing concrete aids that are useful in sharpening visual acuity. The Stern materials by Houghton Mifflin, Teaching Devices by Winston, and Arithmetic Tangibles by Creative Playthings represent a few sources providing this type of assistance.

A portion of the suggested aids are superficial, but many, fortunately, contain worthwhile hints for teachers concerned with concept-building in elementary school mathematics, develop the deeper aspects of numbers, suggest ways to uncover the hidden learnings, give valuable back-ground information for the teacher, explain games of many kinds, and help provide for the individual differences found in the average classroom.

A compiling of the contents of some written materials shows the trend taken by writers and publishers and their consideration of problems that confront teachers and pupils. How can the teacher begin to profit by the helps that surround her? Which ones will she choose? How can she evaluate them?

The majority of materials analyzed by the writer have one important idea in common, that the child, with proper guidance and many practical experiences, should discover the basic concepts for himself. Since learning is thinking, the child needs to have practical examples to think about; situations close to his pattern of thinking and living. The child should find out the possibilities and impossibilities of number manipulation through rational counting and working with arithmetical devices designed for his level of achievement either by the teacher, by commercial firms specializing in such equipment, and sometimes through the child's own inventiveness.

The teacher's first step could be to examine the basic textbooks being used in her own system. While reading the teachers' manuals adjacent to her grade takes some time and effort, the results will prove satisfying. If she is a second grade teacher, she will want to become familiar with the contents of the first and third grade manuals. In this way she will be gaining insight into the methods of presentation of materials as well as the accompanying philosophy. She needs this basic background in order to meet the varying requirements of children, for she will have a wide range of abilities in her room. She will be surprised, too, at the change in concepts she will witness in her own thinking. This is good, for her children will reap the harvesting of her ideas.

Following familiarity with the basic texts, she should want to discover what other textbooks offer on her grade level. Let us examine some teachers' manuals together to find the treasure stored therein. There is no attempt in this examination to compare books for all have worthwhile features. Let our purpose be one of finding good practices that we can adapt to our situations. The more we have, the better the quality of learning will be.

Investigating Sources

The writer presents this cross-section of sources for consideration. The ideas extracted from these books suggests only a preview of the wealth of materials one may find.

Row-Peterson's *Arithmetic Manual, Book One* (1952; Wheat, Kauffman, Douglas) covers primer and grade one. Divided into three parts, the book contains the arithmetic to be taught and suggestions for using the book.

A good explanation of number names is found here. Eleven was derived from *ein lif*, meaning one and ten, then einlif, finally eleven. Twelve comes from *zwa* und *lif*, meaning two and ten, then zwalif, and now twelve. The teens, 13–19, are obvious—3 and 10, 4 and 10, etc. Twenty was once twain tens, then twenty.

The introduction gives the teacher a sound foundation in understanding the number system. The degree of her understanding determines the degree of success her pupils will enjoy in working with numbers.

Suggestions for supplementary activities in the primer include number charts, counting of objects, giving directions using numbers, feeding animals—how many peanuts shall we feed the squirrels? and comparing groups.

This manual leaves the field of incidental learning to the teacher who is alert to use all opportunities to further number knowledge.

The section entitled "Suggestions for Using Book One" mainly concerns the adding and subtracting of groups and helps for the teacher in presenting materials. Again, the teacher presents the enrichment. Making up problems and making number cards are two of many activities.

D. C. Heath's *Learing to Use Arithmetic, Book 2.* (1958; Gunderson, Hollister) stresses the learnings achieved by the child through manipulative devices, observations, discussions, and experimentation. It gives ideas on grouping for instruction, providing for individual differences, ways of evaluating progress, and logical procedures in presenting things coincident with the pupil's age and experience.

Activities are numerous: cutting out pictures for number charts, worksheets for practice in associating numbers and number names, retelling stories involving comparisons like "Jack and the Beanstalk," showing and discussing coin collections and making coin displays, playing cafeteria with cards of pictured foods, making up problems using numbers only, relating pints and quarts to liquids—milk, paint, oil, using pictures showing comparisons, stringing different colored beads by 2's and 3's, constructing paper plate clocks, reporting on highway sign symbols, finding stories and rhymes that contain numbers in titles or in content, illustrating numbers, cutting numbers for fractions, making charts showing what can be bought for varying amounts of money, filling out dot diagrams, illustrating number families, learning the rudiments of Roman Numerals, checking answers with counters arranged in bundles or stacks of ten each, making strips of pictures with combinations on back, a number family mural.

World Book's *Growth in Arithmetic, Grade 3* (1957; Clark, Junge, and Moser) contains interesting self-directing sections such as "Just for Fun," "Be Your Own Teacher," and "For Faster Learners." In problem-solving this text's major objective is the solving of problems common to the child's experience.

For faster learners one can find these activities: price lists—looking up the prices of children's costs, hats, dresses, slacks, and

making up problems about them; compiling lists of children's magazines in the school library; party-planning; numbers in reading —as the temperature in Mexico; telling time with modern clocks (no numerals).

For slower learners: making geometric designs in connection with fractions gives meaning and creates interest; helping the child who finds writing numbers difficult is accomplished by having him dip his pointing finger in a paper cup of water and trace the figures the teacher has made. Tracing dot figures also helps. In measuring, the use of hand spans, pencil lengths, arm lengths, string as well as rulers and yardsticks will enable the slow child to grasp distance concepts. Discuss early types of measuring. It's fun to know how and why something began.

Other activities are preparing blackboard materials, investigating the uses of a thermometer—thermostat, cooking thermometer, medical thermometer. Discussing the types of clothing worn at different temperatures would fit in here.

"Be Your Own Teacher" presents problems related to child life. Reasonable guessing is expected. Fast-moving pupils could secure prices from the school cafeteria and work out the cost of a good lunch. Pupils are challenged to solve problems before explanations are given.

Scott Foresman's *Seeing Through Arithmetic, Grade 4* (1956; Hartung, Van Engen, Knowles) states that in problem-solving the pupil is helped to recognize what is happening the problem-situation, is taught to describe the situation by using arithmetic symbols, is shown how to select the proper arithmetic process to solve the equation. According to this manual, the pupil must be able to recognize the type of action that takes place.

The Expanded Notes sections have enriched lesson plans. Suggestions include: measuring inches, feet, yards; discovering how these units were decided upon as standard measurements; and finding out if these units of measure are universally accepted and used.

One interesting device is making a scale in ounces. A 4×12-inch piece of heavy cardboard, a paper clip, and a rubber band are the only requirements. Loop the rubber band through the hole. Attack to the rubber band the paper clip which has been straightened out except for one bend. Turn the opposite end to form a hook for holding objects. Let the children calibrate the scale as far as possible by testing objects of known weight.

Pasting grocery ads on cards helps the child to "shop" and show his purchases in combinations on paper. Making lists of things showing fractions they have observed —recipes, measuring cups, half-pound cartons, etc., keeps this topic practical.

Other games are described. One calls for several corks and a can. To play "Cork Drops" paint numbers on the ends of the corks. Try to drop the corks into a can from a specified height. Add up the score according to the numbers on the corks that fell into the can.

Laidlaw's *Understanding Arithmetic, Grade 5* (1956; McSwain, Ulrich, Cook) develops patterns of reading, observing, thinking, and reasoning that will lead the child toward an understanding of the meaning and use of the number system. There is a provision for individual differences, and there are pages for self-evaluation entitled "To Help You Remember."

Suggestions include using road maps, time tables, air-routes; the spending of school funds; the cost of textbooks for a class; personal budgets; scale drawings of the schoolyard, football field or baseball diamond.

The first part of the manual has lesson plans. The book has a section on the arithmetical meanings that children need to understand; decimal number system, process of counting, the four fundamental processes, common fractions, decimal fractions. There are also chapter tests.

Macmillan's *Making Arithmetic Clear, Grade 6* (1957; Pepper, Carpenter) has a built-in readiness feature as a part of each develop-

mental lesson as well as readiness activities. There is extensive provision for individual needs—optional starred material, supplementary practice exercises and tests in both text and manual. There is an expanded section on reading in arithmetic, problem-solving, the testing program, classroom equipment, and vocabulary.

For fast learners a history of counting could include making an abacus and demonstrating the use of the abacus. In developing Roman Numerals a sketch of a sun dial using Roman Numerals may lead into ancient and modern methods of telling time. Reporting on the United States Bureau of Standards in connection with measuring, finding the melting point of iron, aluminum, gold, and silver, and comparing distances in miles are worthwhile activities.

Scribner's *Arithmetic, Grade 7* (1957; Gager, Echols, Madden, Shuster, Kokomoor) states that both the teacher and the pupil must have patience to gain mastery of the instrument.

The introduction includes the reasons for studying math, how children learn, the materials needed in the classroom, and a portion on problem-solving. Part 2 of the manual deals with teaching suggestions, including such topics as graphs, percentage, simple geometry.

Teachers are encouraged to guide pupils into investigation and research, to broaden the scope through intensive study. The primary reason for studying math is to learn how to solve quantitative problems with accuracy and reasonable speed.

Learning processes involve activities which give pupils concepts that in turn will enable them to use the abstract symbols of math in various operations. Much opportunity for experimentation and observation are provided. Testing conclusions and sharing knowledge are suggested to further the pupils' learning potential.

The concept of decimal place-value is developed through a demonstration of the abacus and a discussion of the number systems of the Mayans and the Babylonians. A project would be making an abacus.

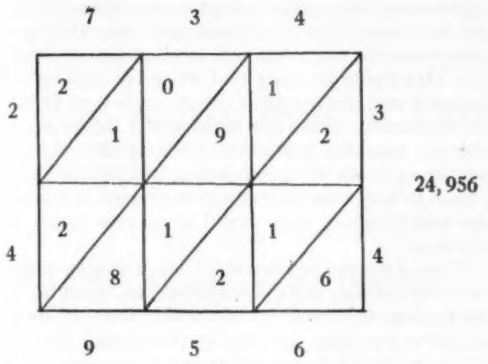
Other things to do are finding ads in newspapers, using references like the World Almanac, the rounding of numbers, and working out magic squares. There is a commendable section, too, on graphs, where degree days are mentioned. Here the students could determine the amount of fuel needed to heat buildings and could make original graphs to tie in with their studies.

One interesting brain teaser is to find a mixed number which when multiplied by 7 will give exactly 23.

The manual also suggests the use of squared paper for studying fractions and percents. Additional activities might be scale drawings, the metric system of measures, original designs employing geometric forms, and the use of circles to show relationships.

Winston's *The New Knowing About Numbers* (1956; Brueckner, Grossnickle, Merton) has for its basic philosophy learning by discovery. That the classroom is a laboratory is evident in the helps this guide gives the teacher. It has suggestions for time allotment and a special section on teaching aids—rounding off numbers, types of ratios, altimeter, constructing perpendiculars, property taxes, etc. The construction of laboratory materials is explained—flannel boards, fraction charts, carpenter's rule.

Students interested in early methods of finding answers might investigate the lattice work multiplication used in Europe during the Middle Ages. In multiplying 734×34 draw a rectangle cut in half horizontally and divided into thirds vertically. Add four diagonal lines as shown below. Place the multiplicand across the top of the rectangle, each digit at the top of the column. Place the multiplier vertically at the right side. Begin to multiply 3×4 and place the product in the first two diagonals at the top right of the rectangle. After multiplying by both figures in the multiplier, find the final product by adding the figures in the diagonal columns.



There are starred problems for enrichment, number "quickies," circle graphs representing data found in other school subjects or in school activities, suggested study of calendar reform—World Calendar as opposed to the Gregorian Calendar, visits to savings banks, and Junior Achievement activities such as forming a corporation.

Broadening Horizons

In addition to teachers' manuals are pamphlets dealing solely with mathematics. The N.E.A. research bulletin, "Teaching Arithmetic" (1953; Ruth Morton) takes up a variety of topics—grade placement, drill, testing, preparation of teachers.

An A.C.E.I. publication, "Arithmetic, Children Use It" (1954; Edwina Deans) considers the experiences of children by age, beginning with the 4's. Many experiences of young children are described—what they encounter and how they attempt to solve these problems. There are hints for the teacher as to how she can guide children to discovering concepts. Excellent concrete examples of life situations are recorded.

The Board of Education of the New York City Schools has issued "Arithmetic, Kindergarten through Grade 3" (1947–48) which begins with the premise that thinking and understanding have priority over number manipulation. This pamphlet insists that problems should be considered in the light of social implications. There are charts showing interrelationships and sequence of numbers and processes at each level. For each level numerical experiences are presented along with the significance of each situation.

Wesleyan University, Middletown, Conn. features arithmetic in two "Curriculum Letters." No. 15 treats "Solving One-Step Problems in Arithmetic" and No. 28 deals with "Arithmetic Tests."

Row-Peterson's monographs include: No. 79, "Growing To Understand Measurement," No. 83, "Teaching Materials as Keys to Understanding Arithmetic in the Primary Grades" (tells what to include in pupils' individual number boxes); No. 84, "Arithmetic Aids For The Middle Grades" (gives directions for constructing interlocking circles for fractions.

Charles Merrill & Co. issues a "Teacher's Memo for Arithmetic." To date, numbers 1–6 are available in quantity.

Another Row-Peterson publication, "Guiding Beginners in Arithmetic," by Amy De Moy (1957) covers arithmetic in grades 1 and 2. There are teaching suggestions, ideas for the presentation of numbers, games for motivation and practice. There is also a section on helping pupils with problem-solving. This author says that the essence of a problem is the question it asks. The book also discusses the vocabulary of comparisons and gives hints for oral and written work. It lists positional and directional words.

It further mentions the mathematical terms with which pupils should become familiar. On this subject consult Gates' "A Reading Vocabulary for the Primary Grades," and Rinsland's "A Basic Vocabulary of Elementary School Children," for additional vocabulary.

Appropriate projects and activities for primary children are explained. Experiments with fractions and measures are suitable for this level.

D. C. Heath's "Teaching Arithmetic in Grades 1 and 2" by Agnes Gunderson and George E. Hollister (1954) includes information needed by the teacher for the number program and presents a program for

these grades. The book stresses the importance of arithmetic in daily life and the need for the school to provide experiences to help pupils meet their needs and solve their problems.

It has excellent background material for the teachers' information as well as teaching principles and practices. Among the teaching materials and devices are subtraction and addition wheels.

The second part of the book deals with the child's readiness to learn and the content that can be presented at the primary level. Particularly outstanding is the basic vocabulary for grades 1 and 2.

R. L. Morton's book, *Teaching Children Arithmetic*, (1953; Silver Burdett) would be helpful to primary, intermediate, and upper grade teachers. The systematic development of each process is presented, level by level. This book goes a step beyond research and psychology by including evidence actually observed in the field. In connection with problem-solving, the author states that here "the teacher has a chance to realize one of the most important goals of arithmetic instruction" (p. 513).

While *How to Teach Arithmetic* by Harry Grove Wheat (1951; Row-Peterson) is designed primarily for college students preparing for teaching, it has enough useful content to have a place on any elementary teacher's reference shelf. Several of the final chapters tell stories of weights and measures, of the struggle to perfect the number system, and methods that early peoples used in counting.

Teachers contribute much to the future by examining and utilizing, wherever possible, all that today has to offer in aids and services. This giving of time and effort is twofold: the teacher is enabled to have increased knowledge and "know-how" at her command, and in turn, she can more adequately encourage the children to take advantage of their number heritage.

EDITOR'S NOTE. Teachers' manuals that accompany textbooks are now being prepared so that they are useful not only to the beginning teacher but also to one who has become well established. The authors of these manuals realize that all good arithmetic cannot be learned from a textbook and hence they give suggestions for discovery and for extension in learning. Miss Foote proposes that we use several sets of manuals and use the ideas that seem best to fit our circumstances. There are many many things available for teaching arithmetic meaningfully. Simple pebbles or beads of various colors or sizes cannot be placed in books but they can be suggested in a manual and they are very useful at certain stages of learning.

When a teacher works with children seeking better modes of learning she too is learning and growing in her profession. This involves not only the arithmetical ideas but also the way pupils think. This of course involves a mental alertness and an open mind instead of an older pattern of education in which the teacher dictated what was to be learned and how it should be learned.

Visual Aid Review

Learn-By-Doing Fraction Kit. Models of Industry, Inc., Berkeley 10, California (1958). Price $6.95 each.

Materials in this kit consist of (1) 10 strips marked to show 1/2 of a unit, 1/3 of a unit, etc. Velour paper is used so that these materials may be used as a fraction chart on a flannel board to demonstrate the properties of fractions, (2) a set of 7 pieces also on velour paper including 1/2 unit, 1/3 of a unit, 1/4, 1/5, 1/6, 1/8, and 1/10. These may be used in developing an understanding of the processes using fractions, (3) a set of ratio and proportion cards, (4) a set of Math Fun cards which contain problems on one side and solutions on the back of each card. These are primarily for purposes of providing interest, and (5) a set of percentage and decimal flash cards.

The materials in this kit are a means of helping develop the idea of what a fraction is including decimal fractions and percentage, the role of the pair of numbers used in the fractional numeral, a study of families of equivalents, the rules for adding, subtracting, multiplying, and dividing fractions, proportions, and the part, whole, per cent relationship.

E. GLENADINE GIBB

A Method of Front-End Arithmetic

ANDRE J. DEBETHUNE
Boston College, Chestnut Hill

AS A TEACHER OF CHEMISTRY, I use arithmetic as a work tool every single day, and so do my students. There is hardly a problem in chemistry whose solution does not call for at least one, if not many, arithmetical operations. The routine handling of large volumes of arithmetical work is an accepted occupational hazard of the chemist's. It is a tribute to our schools that our chemistry students come to us well prepared in this regard. Our budding chemists do know how to add, subtract, multiply and divide, and they can do so without hesitation or dismay.

Chemistry courses also normally provide practical drill in the use of such computational short cuts as the slide rule and logarithm tables. These two useful tools do have their limitations however. They neither add nor subtract. They simplify multiplication and division, especially long sequences of these operations, but at the price of a limitation in accuracy. The ordinary ten inch slide rule is accurate only for the first three figures of the answer; four place logarithms for four figures; five place logarithms, for five figures, and so on. Instances do occur where the short cuts fail to give the desired accuracy. The chemistry student must then fall back on arithmetic. Even when his laboratory is equipped with an electric calculator to grind out his computational drudgery for him, the electric motor is still operating according to the rules of arithmetic.

Classically, arithmetic builds up numbers from the number one. By successive addition of unity, these are built up in sequence: two, three, four . . . up to nine. Nine plus one makes ten, which is written 10 in the decimal system (and, curiously enough, "1010" in the binary system). Numbers are then counted in tens and ones up to 99. One plus that makes one hundred, 100. Numbers are then counted in hundreds, tens, and ones up to 999, *et cetera.* It is logical in this system to operate on the ones first, then on the tens, then on the hundreds, and so on. This approach gives rise to our classical "rear end" method of addition, subtraction, and multiplication.

"Rear end arithmetic," besides being customary, is also convenient where space is at a premium, i.e., in tax blanks and in business forms. But it can be an inefficient method of computation. The important figures in an answer are those at the front end, and these, rear-end arithmetic attains last of all. In rear-end multiplication, much effort is expended in obtaining first figures that are often later discarded as useless. For example, the annual interest on a principal of \$96.95 at $3\frac{1}{4}\%$ is computed as \$3.150875. The first three figures attained the "5," and the "7" and the "8" are all later to be discarded. The "0" serves merely as a guide in rounding off to the nearest whole number of cents and the answer is finally taken as \$3.15, i.e., in terms of the last three figures to be reached by the usual rear-end method. In addition and subtraction, the chemist often deals with quantities having unequal numbers of decimal places. For example, let him find the formula weight of erbium sulfate, $Er_2(SO_4)_3$, from the atomic weights: Er 167.2, S 32.066, and O 16.0000. Twice 167.2 plus thrice 32.066 plus twelve times 16.0000 add up to 622.5980. Yet the result is not valid beyond the first decimal place because of the limited accuracy in the atomic weight of erbium. The answer should be reported as 622.6. Three useless decimal places have been computed first, later to be discarded.

In an attempt to meet these difficulties

my students and I have been experimenting with "front-end arithmetic." A short account of this method has already been published elsewhere (*Columbia University Forum*, Volume 1, Number 2, p. 25, Spring 1958). To a person accustomed to the usual rearend methods, front-end arithmetic may appear strange and unfamiliar at first. It takes a little practice to acquire a "feel" for it. As this feel develops, some of its advantages become apparent. For instance, figures that are to be discarded are eliminated before computation. Even where no discards are permitted, front-end arithmetic provides a "convergent" approach to the answer, i.e., an approximate value for the answer becomes apparent as soon as one or two columns of the front-end have been operated on, and long before the entire operation is finished. As successive columns are included in the result, the changes in the partial answer already attained become progressively smaller. In contrast, the "divergent" approach of rear-end arithmetic builds up the least significant numbers first, e.g., the cents and dimes, and only after a prolonged bout of figuring gives the significant digits, e.g., the dollars, tens, hundreds, thousands, . . . of dollars!

Front-end arithmetic transfers computational fatigue from the most significant to the least significant columns. Furthermore, it can be made to eliminate carry-overs and their errors. And by laying out in plain sight the structure of the operation in its component parts, it makes errors and mistakes easier to ferret out and to correct.

But front-end arithmetic does consume more paper! Since becoming addicted to this method, I carry a scratch-pad with me at all times to do my front-end operations on. In my check-book, the bank allows me only enough room for rear-end subtraction, and here I yield to the necessity of returning to rear-end methods. Actually a mastery of front-end arithmetic does not preclude concurrent usage of the rear-end method, and one method can be used to advantage as a check on the other. Since beginning work with front-end arithmetic, my computational speed has increased to such a point that I often do the arithmetic now instead of hunting for the electric calculator around the lab.

Front-end arithmetic is not anything really new. Many business men have resorted to front-end techniques, in one form or another, to get significant answers more quickly. My oilman regularly computes my bill, for a fuel-oil delivery, by a technique of front-end multiplication of his own devising. The approach to numbers from the front end, in the teaching of arithmetic, has been convincingly discussed in these pages by Professor J. Allen Hickerson (ARITHMETIC TEACHER, V, 178 (1958)) of the New Haven State Teachers College, New Haven, Connecticut. The front-end methods described below represent techniques that the writer and his students have found convenient and practical. Other and better techniques may yet be devised. Those given here are offered as suggestions for an interesting variation to the conventional rear-end approach.

Front-End Addition

The numbers are lined up as usual in their proper vertical columns (Fig. 1). The first column is totalled and the subtotal:

```
$37.55
 63.86
 97.23
  8.98
-------
(18      )
(  25.   )
(   2.4  )
(    .22 )
-------
(1 _7.62 )
(10      )
-------
207.62
```

Fig. 1

$3+6+9=18$ written in its proper place under the line. At this stage we already know that the final sum will exceed \$180. The second column is summed, its subtotal: $7+3+7+8=25$ is written in its proper place under the line. The first two subtotals add up to \$205, a second approximation to the final answer. The third column subtotal: 24, is again written in its proper place under

the line. The first three subtotals give the partial sum \$207.4, a third approximation to the final answer. The fourth column subtotal: 22, is again written in its proper place under the line. Added to the previous partial sum, it yields the final answer \$207.62.

The four column subtotals as they appear under the line can also be added by rear-end addition at this stage to yield the sum \$207.62. Or a second round of front-end subtotals can be obtained: 1, 10, 7, 6, and 2 and written in their proper places under a second line (Fig. 1). This second round of column subtotals can then be added, front-end or rear-end, to yield once again the final answer \$207.62.

This same addition, worked out from the rear-end, yields the following partial sums: after one column, 22 cents; after two columns, \$2.62; after three columns, \$27.62, and then the final answer \$207.62. In contrast, at comparable stages of progress, front-end arithmetic yields the partial sums: \$180, \$205, \$207.40 and finally \$207.62, a much more "convergent" approach to the desired result.

Front-end arithmetic can also be used to advantage in summing numbers that are not lined up vertically (Fig. 2). The numbers in the hundreds place are summed first, and

\$275.33, 62.18, 59.95, 177.33

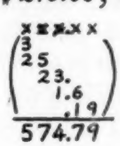


Fig. 2

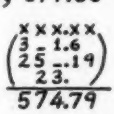


Fig. 3

their total 3 written in the hundreds place. The tens add up to 25 and this number is written on the next line in its proper place. The partial sum at this stage is \$550. The units add up to 23, this is written on the next line in its proper place, and the partial total is now \$573. The tenths add up to 16 and this is written in its proper place, the partial total now is \$574.6. Finally the hundredths add up to 19, this is written in its proper place and the final total is \$574.79. The column subtotals can also be added, front-end or rear-end, to yield once again \$574.79.

Figure 3 illustrates a repetition of this sum by front-end arithmetic but with the column subtotals entered in such a way as to save space.

Front-End Subtraction

Line up the numbers as usual (Fig. 4). Find the algebraic sum of each column, starting at the front end, and write the column subtotals under the line in their proper places: 1, then -2, which is conveniently written, $\bar{2}$, then $\bar{1}$, then 2, then $\bar{2}$. The resultant number $5\bar{2}\bar{1}.2\bar{2}$ is a hybrid of positive and negative quantities which stands for $500-20-1+0.2-0.02$. This can be resolved by mental arithmetic to yield successively 500, 480, 479, 472, 479.2, 479.18, the final answer. It can also be resolved with pencil and paper, as follows: Take the grouping $5\bar{2}$. This stands for $50-2$ or 48. Write 48 under the $5\bar{2}$. The next two digits are then $8\bar{1}$, which stands for $80-1$ or 79. Write 79 in the next line in its proper place. Finally, the grouping $2\bar{2}$ stands for $20-2=18$. Write 18 under $2\bar{2}$. Now transpose the bottom digits of each column, i.e., 4, 7, 9, 1, and 8, under the second line, and this is the answer: 479.18.

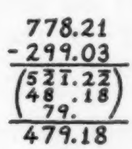


Fig. 4

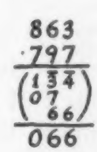


Fig. 5

Figure 5 illustrates a pitfall to be avoided in front-end subtraction. The algebraic column subtotals yield the hybrid $1\overline{34}$ which is, of course, $100-30-4=70-4=66$. In the paper and pencil method, take the grouping $1\bar{3}$. This stands for $10-3=07$. Write 07 under the $1\bar{3}$, not forgetting the 0 which is very important. The next two digits $7\bar{4}=70-4=66$. Write 66 on the next line in its proper place. Transposition of the bottom digits gives 066, or 66, the right answer.

Algebraic Summation

68.33, -22.95, 18.08, -41.67

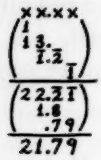


Fig. 6

15.92, -3.16, -14.59

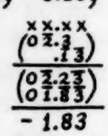


Fig. 7

To find the sum of credits and debits, without segregating them (Fig. 6). The algebraic sum of the tens is $6-2+1-4=1$ and is written in the tens place. The units add up to 13, this is written on the next line in its proper place. The tenths add up to -12. This is written as $\overline{12}$ on the next line in its proper place. The hundredths add up to -1 or $\overline{1}$. This is written on the next line in its proper place. Summation of the columns yields the hybrid $22.\overline{21}$ which is $+20+2-0.2-0.01$. This gives, successively $+20$, $+22$, $+21.8$, and finally, the answer $+21.79$, i.e., a credit of 21.79, or 21.79 "in the black!" The hybrid can also be resolved as indicated in Figure 6: the grouping $2\overline{2}=20-2=18$, then $8\overline{1}-1=79$. The final answer, as given by the bottom digits, once again is $+21.79$.

Suppose the debits exceed the credits (Fig. 7). The column subtotals, starting at the front end, are: 0, $\overline{2}$, 3, and $\overline{13}$ and these are written in their proper places. Further summation yields the hybrid: $0\overline{2}.2\overline{3}$. This is $-2+0.2-0.03$ and can be readily resolved to yield, successively, -2, -1.8, and finally -1.83. Or else, the grouping $\overline{2}2$ $(=-20+2=-18=\overline{18})$ is replaced by $\overline{18}$ and the final result is $0\overline{1}.\overline{83}$ or -1.83, i.e., 1.83 "in the red!" This method can also be used even when it is not known in advance whether the total will come out "in the black" or "in the red."

Front-End Multiplication with a Guide Line

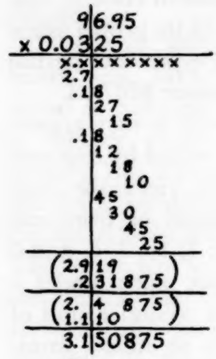


Fig. 8

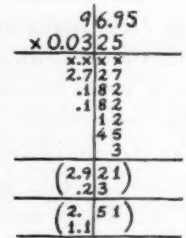


Fig. 9

To find $3\frac{1}{4}\%$ of \$96.95, i.e., to multiply it by 0.0325. Line up the two factors by their front digits (Fig. 8), and draw a vertical guide line after the front digits. The decimal point is located by counting places right and left of the guide line, i.e., *one place to the right for the multiplicand plus two places to the left for the multiplier* locates the decimal point *one place to the left of the guide line for the product.* Start multiplying from the front end, writing the subproducts in their proper places: $3\times9=27$, $3\times6=18$, $3\times9=27$, $3\times5=15$, then $2\times9=18$, $2\times6=12$, etc., until every digit of the multiplicand and the multiplier have been combined into a subproduct. To locate the proper place of a subproduct, count places *to the right of the guide line*, e.g. the subproduct $5\times6=30$ occurs with its last digit in the *third place to the right* of the guide line since *three* is the sum of *two places to the right* for the 5 *plus one place to the right* for the 6. The subproducts can be summed by either front-end or rear-end methods. The front-end method yields the column subtotals 2, 9, 23, 19, 18, 7 and 5. These have been written in Figure 8 on only two lines to save space. The next round of column subtotals, from the front end, yields 2, 11, 4, 10, 8, 7, and 5, written also on only two lines. The next set of subtotals finally yields the complete answer as \$3.150875 without any discarded digits.

Figure 9 gives a repetition of the front-end

multiplication of Figure 8, carried out so as to yield an answer to the nearest cent, i.e., the multiplication is cut off after the third (or tenths of a cent) decimal place., i.e., after two places beyond the guide line. Note that the initial subproduct $3 \times 9 = 27$ is written immediately to the left of the guide line. The next subproduct $3 \times 6 = 18$ is written one place to the right of the guide line. The next subproduct $3 \times 9 = 27$ occurs two places to the right of the guide line and may be written next to the first 27 to save space. The next subproduct $3 \times 5 = 15$ is rounded off to a 2 in the third decimal place and written next to the 18 to save space. Then the $2 \times 9 =$ a second 18 is written one place to the right of the guide line; the $2 \times 6 = 12$ occurs two places to the right of the guide line, then the second $2 \times 9 =$ a third 18 is rounded off to a 2 in the third decimal place and written next to the second 18 to save space. The next subproduct $2 \times 5 = 10$ falls entirely beyond the third decimal place and may be ignored. Next $5 \times 9 = 45$ is written two places to the right of the guide line, then $5 \times 6 = 30$ contributes a 3 to the third decimal place and all further subproducts may be ignored. The column subtotals, from the front end, are 2, 9, 23, and 21, and are written on two lines only, to save space. A second round of subtotals gives, from the front end, 2, 11, 5 and 1. The third round of subtotals gives answer 3.151, or to the nearest cent, $3.15.

The occurrence of *zeros* in a factor must be carefully watched. Zeros contribute no subproducts, but they do affect the location of subproducts in their proper places. By counting the proper number of places *to the right* of the guide line, subproducts can be easily located. In Figure 10, the decimal point is located *one plus one* or *two places* to the right of the guide line. The subproducts: 18, 9, 30 and 15 occur, respectively, to the left of, and two, two and four places to the right of the guide line. With a guide line, the initial subproduct (e.g., $3 \times 6 = 18$ in Figure 10) occurs always immediately to the left of the guide line. Further subproducts occur in the

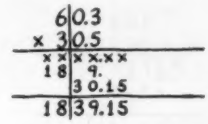


Fig. 10

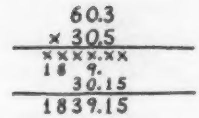


Fig. 11

proper number of places to the right of the guide line, as indicated above.

Front-End Multiplication without a Guide Line

A guide line is not strictly necessary, and subproducts can still be placed properly by reference to the decimal point only. Places to the right of the decimal point can be counted off in the usual way. But the calculator should be on his guard for the places of numbers to the left of the decimal point. Numbers immediately to the left of a decimal point should be always counted as being in the *zeroth place*. Numbers in the tens place should then be counted as *zero-one places* to the left of the decimal point; numbers in the hundreds place, as *zero-one-two places* to the left and so on.

Figure 11 illustrates this rule by repeating the front-end multiplication of Figure 10 without a guide line. The 6 and the 3 are both *zero-one places* to the left of the decimal point. *One plus one* make *two* and the subproduct 18 is written *zero-one-two places* to the left of the decimal point. Next the subproduct $3 \times 3 = 9$ should be placed *zero-one to the left plus one to the right* equals *zero places* to the left or immediately to the left of the decimal point. Similarly the product $5 \times 6 = 30$ occurs *one to the right* plus *zero-one to the left* equals *zero places* to the left. The final subproduct $5 \times 3 = 15$ occurs *one to the right* plus *one to the right* equals *two places to the right* of the decimal point.

Figures 12 and 13 illustrate another front-end multiplication carried out both with, and without, a guide line. Subproducts are located by reference to the guide line in Fig. 12, and by reference to the decimal point in Fig. 13, by use of the *zero-one-two* . . . rule for places to the left of the decimal point,

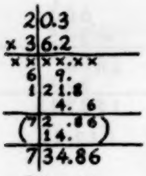


Fig. 12

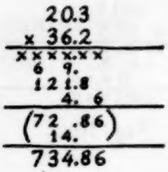


Fig. 13

and the *one-two* . . . rule for places to the right of the decimal point.

The reason why two separate rules are needed, in dealing with places to the left, and to the right, of the decimal point has been admirably discussed by Dr. Francis J. Mueller of State Teachers College, Towson, Maryland, in his paper "The Neglected Role of the Decimal Point," (ARITHMETIC TEACHER, V, 87 (1958)). Dr. Mueller points out that place numberings are symmetrical, not with respect to the decimal point, but with respect to the units' place. If the *units' place* is given *place number zero*, the *tens'* and *tenths' places* are then given the *place number one*, to the left, and to the right, respectively. Similarly, the *hundreds'* and *hundredths'* places are given the *place number two*, to the left, and to the right, respectively; the *thousands'* and *thousandths' places* are given the *place number three*, to the left and to the right, respectively. The reader will readily correlate these place numbers with the powers of ten in the sequence: . . . $1000=10^3$, $100=10^2$, $10=10^1$, $1=10^0$, $0.1=10^{-1}$, $0.01=10^{-2}$, $0.001=10^{-3}$, ... The counting off of places in multiplication, is equivalent to the algebraic rule that $10^a \times 10^b = 10^{(a+b)}$.

With a guide line, however, the leading subproduct occurs immediately to the left of the guide line. All other subproducts are located by use of the *one-two* . . . rule for places to the right of the guide line. The decimal point is located by use of the *one-two* . . . rule for places, both left and right of the guide line.

Division

Division as usually done is already a front-end operation. Since division is repeated subtraction, a rear-end technique of division could be worked out, but "rear-end long division" would be painfully long indeed! This can occur with an electric calculator, when an automatic division is begun with the front end of the divisor to the right of the front end of the dividend. The electric motor grinds on, and on, endlessly subtracting, chipping away at the dividend little by little and hoping eventually to catch up to it. The "STOP" button is mercifully present, to bring a halt to the motor's agony. The numbers can be punched into the calculator again, and the automatic division repeated with the two front ends properly lined up.

Short division is basically a front-end operation as it stands. Long division is a front-end operation also, but involves pencil and paper subtractions which can be done by either front-end or rear-end methods. A tricky problem in long division is to find how many times the divisor "goes into" the dividend and into each successive remainder. This can be done by shrewd guess-work, by trial and error multiplication, or by successive subtraction. This latter method seems appealing, for a multi-digit divisor, since it reduces the whole task of long division to nothing more than a sequence of subtractions. In Figure 14, the division of 627.456 by 24.32 is carried out. The decimal point of the quotient is located in the standard way (Fig. 14b). Next, a table of multiples of the divisor (Fig. 14a) is prepared by starting from ten times the divisor $=10\times 24.32 = 243.2$ and subtracting the divisor eight successive times. The successive subtractions are carried out in Figure 14a from the front end, and illustrate certain further short-cuts made possible by handling two columns at once. For example, in going from the 9th to the 8th multiple of the divisor: $21-2$ can be written immediately as 19 without going through the intermediate hybrid $2\bar{1}$. In going from the 7th to the 6th multiple, $170-24$ can be written directly as 146 without going through the hybrid $15\bar{4}$, and $24-32$ can be written directly as $0\bar{8}$. In going from the 5th to the 4th multiple, $121-24$ is written directly as 97 without going through the hybrid $10\bar{3}$. The eighth subtraction yields twice the divisor, and the divisor itself can then be entered to complete the table. Or else, a

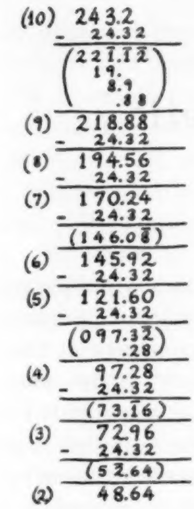


Fig. 14a

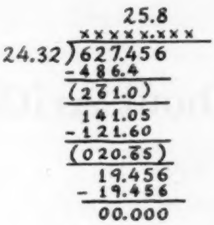


Fig. 14b

ninth subtraction may be performed, which should yield the divisor once again, as a check on the first eight subtractions.

The table of multiples (14a) can then be used, in the long division itself (14b), to choose the appropriate multiple to subtract from the dividend, and from each one of its successive remainders, until the final remainder becomes either zero or negligible. In Figure 14b, the successive subtractions are carried out from the front-end of the dividend, as is customary in long division, but only here they are carried out by front-end subtraction: the hybrids $2\bar{6}$ and $20\bar{6}$ become 14 and 194, respectively. In all other respects, this long division is normal, and it would have been completely so had rear-end subtractions been used.

The preparation of a table of multiples of the divisor (Figure 14a) can also be done by any appropriate combination of eight operations, whether they be additions, subtractions, multiplications, or short divisions. For example, let the divisor be x. Then $10x-x=9x$, $\div 3=3x$, $\times 2=6x$; $9x-x=8x$, $\div 2=4x$, $\div 2=2x$; $8x-x=7x$; $10x\div 2=5x$. The eight operations suggested replace the eight successive subtractions used above, by three subtractions ($10x$ to $9x$ to $8x$ to $7x$), four short divisions and one simple multiplication (by 2), a method which introduces greater variety, and serves to alleviate dullness or fatigue. These four sequences of operations can be most easily cross-checked by verifying that $5x+x=6x$ and $5x+2x=7x$.

Summary

The methods of front-end arithmetic suggested here provide an interesting variation for, and supplement to, the classical rear-end approach. The rear-end methods are still those which are well established, universally understood, and most economical of space. To someone who is already a skilled calculator with rear-end arithmetic, front-end arithmetic offers a stimulating challenge in speed, in versatility, in accuracy also. By eliminating carry-overs, and by laying out in plain sight the skeletal structure of the operation, front-end arithmetic makes it easier to ferret out mistakes—mistakes that occur frequently with multiple figures, with long sums, etc. These same mistakes often remain concealed in the rear-end method, and are much harder to dig out. Front-end arithmetic provides a convergent approach to the answer, it gives the operator a "feel" for his answer long before the answer is completely worked out. It puts off computational fatigue to the less important columns. On the whole, it provides the calculator with a challenging alternative path to the same final result, and the opportunity for reviving interest in what, to many, has become the deadly and dull routine of arithmetic. Finally, since all numerical work should always be checked for accuracy, front-end and rear-end methods provide alternative paths for the checking of results. Working with numbers has been a particular activity of man since earliest times. Front-end arithmetic provides us with new methods for carrying out a perennial human task, a task which the Lord already gave to Moses: "Take yee the summe. . . ." (Numbers *1*, 2)

EDITOR'S NOTE. At first glance many teachers may be disturbed by the "front-end" approach to computation with whole numbers and decimal fractions. This is a bit like asking certain groups of people to change to "daylight" time during the summer months. They tend to accept the old standard time as something almost divinely ordained. Similarly,

(*Continued on page 32*)

Thinking Afresh about Arithmetic

Caleb Gattegno
London, England

When so many people have already been engaged in examining the basis of our knowledge of number, there seems to be little chance of finding something new to say on that venerable subject. Still it is my contention that we have barely started examining that field and that at least one generation of research workers will be needed to clear the ground that I shall survey in this article.

The critical study of the conceptions of number put into circulation in the last seventy years by Frege, B. Russell, A. N. Whitehead, Hilbert, etc. and more recently by Piaget, may be left for another occasion and perhaps to other writers. It may be that the main difficulty has been the attempt to define *number*, thus assuming that such a concept existed and could embrace all numbers. Are we not there facing a difficulty comparable to the definition of man? Perhaps it would be easier to define each number and to use in the definition some pattern of thought, some process, akin to the complexity encountered. For teachers of elementary grades, that means that we have to ask ourselves whether *counting* is a good basis for the presentation of number and its formation. Counting sometimes means reference to a temporal sequence of sounds often repeated and memorized; sometimes it means (in counting objects) that we recognize that we pass from one sound to the next adding one (assumed to be one and the same unit); sometimes, it means that we can label in a particular way our attribute of a collection which then appears as of the same level of abstraction as other attributes, such as shape, color, consistency, etc.

So long as we hold that we *must* use counting in the formation of numbers, we cannot conceive of any other means of introducing arithmetic to our very young pupils. It has been long recognized, by individual parents and psychiatrists, that anxiety appears in children when they fail to comprehend arithmetic, and that mathematics is the school subject that creates the greatest number of passionate oppositions, these lasting very often through life. Are we not to learn our lesson about the nature of mathematics, of children, and of learning, from this deep emotional involvement of people with mathematics? I maintain that learning mathematics is a most natural activity, comparable to fundamental biological activities such as walking, talking, driving, etc. It is concerned not with knowledge related to memory, but to knowing how and to biological organization linked with reflexes. By making mathematics dependent on memory as we do through counting, tables, rules, we denaturalize mathematics and force the child to meet reality clad in a garment that does not belong to reality and does not fit it.

Do we appreciate adequately the aptitude children must have in order to learn (so early!) to make the sounds people around them make and soon use them freely to express what they want? In learning so quickly to talk so well and so freely, children show us what they can do with concepts. They certainly learn early to suspend their judgement before deciding which object or image belongs to a class they are considering, thus demonstrating that they are *thinking* in classes even while *perceiving* individual objects. They can be said to meet first the general, and by growing in experience, then to fuse into one entity the general structures so that they form the particular classes or objects.

This is a most important observation and should become better known for, when better understood, it will change our outlook on

children and on education. Once made, it leads us to a multitude of improvements of techniques of teaching and to saving of precious time, and time is all we have in life. We should not throw it away as we now so often do, particularly our pupils'. Speeding up the learning processes is not a fancy of one educator. If it can be done by reorganizing our resources and not by cramming, it means we have better understood part of nature, of reality, and can wholesomely serve the purpose of growth, which is the aim of education. If we have not thought in that way we should begin now.

Another observation of importance is that, for everyone of us, experience is one and takes place in the total system which is our self, our body-mind-spirit, and that we can use this observation to integrate all our knowing powers. To say mathematics is concerned with intellectual rigor, is to forget that we feel mathematics as well as think it; that it often is the awareness of something *within us* that is the origin of our awareness of a concept which *we* utter as words or explain, using *our* images. All mathematical discoveries of importance can be traced to a dynamic alteration within our mind of existing organized images, or ideas.

So it is wrong to separate experiences into compartments and say mathematics is abstract, as if music or talking were less, or mathematics is rational (meaning by that that it must be formalized before it gains its status) as if reason had a garment of predilection and did not work in law, cooking or any other human endeavor?

Since emotions can block, they can also motivate and assist. Since children have to mobilize the whole self over and over, in order to organize the new and transient in that which they meet all the time, we must make use of their affectivity in our presentation of mathematical experience. We must use all the components of self (whether known to us or only intuitively suspected) to help them enter as wide a field as possible by putting to them an ever changing field of experience that challenges them incessantly. Mathematics has not disappeared from the surface of the earth, because it is being ever renewed, as are all human arts, by the constant interference of the mind through a new point of view. That is proof that we can make use of every-changing situations to challenge our pupils. But change of viewpoint does not mean by necessity change of material. Even today the set of real numbers remains the most investigated field of study. In fact, in mathematics the approach is not discovery of what is hidden, but the introduction of new lights that put into evidence what was not suspected. The lighting is created by minds and it carries with it possibilities of structuring reality differently. So mathematics is a constantly renewed dialogue that goes back to the same entities to show that they *also* contain this or that.

We say $1, 2, 3, \cdots, n, \cdots$ is the set of integers. It seems such a simple set, but only because we look at its first few members; it is n that hides the mysteries. Who can conceive a number of 10,000 digits? The day we develop that aptitude of seeing large numbers it will not be a discovery but a revolution in number theory that will take place. Our studies are intimately reflecting our stand-points, and we cannot yet say that the building we call mathematics has this or that architecture. Our children, to be educated, must learn to become aware that as they grow on, their horizon varies, widens, their powers increase and with them the field they can encompass and upon which *they* can act.

Our traditional arithmetic is in all its aspects paralyzing, uninspired, therefore pedagogically wrong. To renew it we must attack ourselves and our conceptions and agree that Frege and Russell may be wrong in their proposal of what is the foundation of mathematics.

In my own life came the shock of finding that Georges Cuisenaire, who knew no mathematics and never read anything about their foundations, was the one who showed us the way of making children transcend the limitations tradition justified by reverence,

refusing to touch them. He used the model of musical experience and from it produced the keyboard on which every child can play not music but variations that *are* mathematics. That he came from an unexpected direction is in conformity with what we know of genius and the lighting they put on things. What no mathematician, logician, philosopher could do, a school-master did and for that I and teachers should for ever be grateful and we should all learn from this the great lesson that since life is never ending and always renewing itself, our education should begin to resemble it through our work and endeavors.

Bibliography

1. "The Cuisenaire Discovery," C. Gattegno, *Maincurrents in Modern Thought*, January, 1958, Vol. 14, No. 3, pp. 51–54.
2. "New Methods of Teaching Arithmetic," C. Gattegno, THE ARITHMETIC TEACHER, April, 1956.
3. "Teacher Reactions . . . " C. Howard, THE ARITHMETIC TEACHER, November, 1957.
4. *Numbers in Color*, A New Method of Teaching Arithmetic in Primary Schools, G. Cuisenaire and C. Gattegno, Heinemann, London, 1954.
5. *Arithmetic with numbers in color*, Gattegno, Vols. 1 to 9.
6. *Teaching Mathematics to Deaf Children*, C. Gattegno, Cuisenaire & Co., Ltd., Reading, England, 1958.
7. *Teaching Mathematics in an Expanding Economy—1*, C. Gattegno, Cuisenaire & Co., Ltd., Reading, England, 1959.

EDITOR'S NOTE. We agree with Dr. Gattegno that learning mathematics is a most natural activity and that we should not denaturalize it by dependence upon memory. There are ways of learning that result in things being remembered but they involve stages of thinking, discovery, and reflection that are very different from memorization of facts and principles that may or may not be understood. The one method of learning tends to produce "mathematical power," insight, and understanding while the other seems to be sterile and static. Dr. Gattegno has noted that children sometimes are unhappy with mathematics and develop emotional blocks and he wishes that we might use emotion to motivate and to assist in learning. He states further that our traditional arithmetic is paralyzing, uninspired and therefore professionally wrong. That may be true in many instances but it would be difficult to sustain in many of the classrooms of this country where children enjoy their work in arithmetic and are achieving a depth of understanding, a facility in working with numbers and practice in analyzing and solving problems in the broadest sense.

Editor's Note—A Method of Front-End Arithmetic

(*Continued from page 29*)

we have used "rear-end" computations so long that we tend to hold sacred these time-honored algorisms. What we want is ease of operation and correct answers or rather answers that give the proper degree of accuracy for our purpose. As professor deBethune points out, many of us use the "front-end" approach when this is more convenient. For example, in the addition of two numbers we may proceed from left to right anticipating "carrying" and in subtraction we likewise can look ahead and anticipate "decomposition" if it is needed. True, our conventional algorisms may be more elegant and more sophisticated by are they really easier to learn? Dr. deBethune asks for the use of the "front-end" method as a supplement. When should this be introduced? Should it be made a "discovery" item for more able pupils or should it be "taught" to all pupils? A little thinking and a knowledge of place value of our number system is required in the "front-end" approach. Our conventional methods are more mechanical and they require less paper. Think of the many many hours that pupils have spent on learning addition, subtraction, and multiplication in grades two to five. Can we spare a little time for an auxiliary method? It might be helpful if many teachers studied the several methods of computation used in the past five centuries and then tried to make an objective study of the ease of learning and of using.

"The guy putting is my mathematics teacher. Wait'll you see how he adds out here!"

Reprinted from The Chicago Tribune

Diagnosing Pupil Needs in Arithmetic

EDWENA MOORE AND DEGROFF PLATTE*
San Diego County Schools, Calif.

TOO MUCH TEACHING is done on the basis of assumptions. We assume that our pupils have certain understandings and that they have mastered certain facts. It is easy, in arithmetic, to assume the logical sequence of learning has brought our pupils to us ready to take the next step up the learning ladder. Too often this assumption gets us into difficulties, and, even more, makes arithmetic a frustrating experience for the pupils. However, knowing that our assumptions are untrue is not enough; we need to know how we can pin down the actual accomplishments of our individual pupils in each area of arithmetic.

This problem recently was faced by a group of seventh and eighth grade arithmetic teachers. They began confidently, expecting to find their answer in a standardized test which could be easily administered, scored, and interpreted. Their discouragement at failing to find the instrument which would give them the information they wanted caused them to raise a very interesting question: "Can we build our own diagnostic tests?"

A truly diagnostic test must cover all subareas of each of the major aspects of arithmetic, and so must be more detailed and comprehensive than an achievement or survey test. The first task was to determine the areas of arithmetic to be tested. The areas finally chosen were (1) Understanding the Number System; (2) Understanding the Meaning of Decimal Fractions; (3) Understanding the Meaning of Common Fractions; (4) Understanding the Relationships among Decimal Fractions, Common Fractions, and Percents; (5) Competence in the Addition, Subtraction, Multiplication, and Division of Whole Numbers; and (6) Competence in Using the Four Processes with Decimal and Common Fractions.

The project was a new experience for everyone involved in it. Everything available on diagnostic testing was read by one or more members of the group; sample items were brought to meetings and discussed; methods of test construction became a major topic. It soon became apparent that one danger to be avoided was that of unrealistic and over-ambitious planning, with its resultant discouragements. As a result, it was agreed to postpone decisions concerning tests of problem solving ability and of understanding of measurement until tests of the first six areas were completed.

The first test, on Understanding the Number System, was constructed slowly and, as understanding increased, with constant revision. Subareas (such as understanding place value, and counting and number sequence) were defined slowly. The selection of items for each subarea was based upon group members' knowledge of arithmetic and upon the writings of authorities in the field. Each item had to be judged as to what it actually tested, according to the best understanding of the group, and as to wording. Each item was worded and rewarded until the group felt it was testing only one subarea, and that in so far as possible all other factors were eliminated. As the first test was one of understanding, no items requiring computation were included. For example, among the items testing "Place Value and the Size of Numbers" were the following:

* The committee that did the large share of work on this project included

Harry Wootters, Chairman	Mrs. Nell Steiner
Clarence Cate	Amos Stokes
John D. Clark	Mrs. Lucy Ward
Marvin R. Matthews	

From the Chula Vista Junior High School and the Sweetwater Union High School District.

5. Which number is larger, "twelve hundred" or "one thousand"?
8. Write the largest three-place number possible.
17. How much larger would 356 be if the 5 were a 7?
24. In the number 7,443, what is the largest possible number of tens?

Item 24, incidentally, was the item on the test which was missed by the most pupils; in a quick check with high school seniors in another district, it was found they, too, were unable to regroup numbers easily and with understanding.

Test items finally selected were measured against four criteria:

1. Does the item contribute to the purposes of the test for which it is planned?
2. Is the item easily understood?
3. Is the item worded so that it cannot be easily misinterpreted?
4. Does the item really test the kind of mathematical thinking the group wants to test?

Another series of criteria were applied to the completed test:

1. Are there items of differing difficulty for each of the subareas tested?
2. Does the test cover all important subareas of the aspect of arithmetic being tested?
3. Can the test be administered in a single class period?
4. Is the test easy to score?
5. Are test results easy to interpret?

The first test, in tentative form, was given to all seventh and eighth grade pupils in the school, more than 800 pupils. Copies of the test then were scored, results were analyzed, and the final selection of items was made. Certain items were reworded to clarify meanings, the order of items was changed to place them more nearly in the order of difficulty for pupils in this school, and some items which were too easy, or which gave information duplicating that supplied by other items, were omitted. It was discovered that a few items were enough for some subareas, such as reading and writing numbers, while for other subareas such as making generalizations the group had much difficulty in building any satisfactory items.

Test results were analyzed by grade level, by class, and by individual performance. It was found there was little difference between seventh and eighth grade pupils' scores on over half the items. This is not surprising, for most of the understandings are taught in the lower elementary grades and may not receive major attention in the upper grades. Rounding numbers, however, is taught in the seventh grade, and the percentage of errors in the eighth grade dropped sharply. There was found, in general, to be little difference between classes within a grade; none was outstandingly better or poorer than were the others.

One of the most interesting results of the test came from teacher-pupil discussion of the test and of individual test items. It became clear, for example, that many items were missed because they had been taught in only one way, such as by the use of clues, and that pupils had been able to work problems without a real grasp of the understandings involved. In general, the teachers felt the test verified their original hunch that they needed to place greater emphasis on building better understandings of underlying principles or generalizations rather than on direct teaching of the arithmetic skills.

In some instances relations between pupils and teachers changed as the pupils began to see some real purpose for their work in arithmetic; hampered by a lack of understanding, many of these pupils had done their work mechanically, as and best they could. As the teachers discussed and planned with their pupils for giving the test and using the results, they found that the boys and girls, relieved of the pressures they usually felt toward tests as hurdles, became interested in what was being done, and that their willingness and ability to take the responsibility for improving their own understanding and performance showed remarkable gains. The change in pupil attitudes was one of the major results of the test.

The procedures used for the first test have been followed in the development of the three subsequent tests of understanding. At present the tests of computation are being completed. It has been a slow process, a process of group education as well as one of pupil education. Group members already recognize that, with their increased knowledge and skill in working with diagnostic tests, they could now build better tests. In fact, discussion of needed revisions already is going on.

Comments made by the teachers give an indication of what the project has meant to them. Although building such tests is not easy, one teacher commented, "This is worth all the time and work we have put in." Another said, "These tests are the best means I have found for revising my teaching in terms of what the pupils really can do, and in building on their understandings and skills." A third teacher said, "This is the best thing that has happened to me in my teaching. Although the project has taken a lot of time and planning, I feel I am not wasting time in my day-to-day work with my classes. I now have some real bases for the work we do." The final remark came from a teacher in another school, who asked, "Why can't we in our school work on these tests? Everyone should have a chance to do something so worth while." As for the pupils, their reaction was well expressed by the boy who said, "I didn't realize before that I didn't understand place value. I'm sure glad we are working on it. Arithmetic is beginning to make sense."

EDITOR'S NOTE. The project of making a good diagnostic test is not any easy one but it is most worthwhile particularly for those who study the content, set up the criteria and scope of the test, and then try to prepare items that will measure understandings at various grade levels and with varing depths of understanding. From this account it is apparent that the job was attacked systematically and seriously. At a later date we hope to have samples of the test items and a report of the results obtained. The observation that it is worthwhile spending some time on basic understandings is most acceptable. It is very difficult to obtain all the information one wants about the learning of pupils from a written test instrument. When test results are supplemented with observation and interview the final conclusions seem more valid and are doubly useful to the teacher. The aim of diagnosing and testing is to enhance instruction and learning. A teacher who works seriously on this aspect of the school program will gain additional insights into both the arithmetic and the process of learning.

"Sets" Aid in Adding Fractions

HERBERT HANNAN
Western Michigan University, Kalamazoo

SOME OF THE CONCEPTS developed in modern mathematics can find immediate application in elementary arithmetic. Perhaps the most usable is the concept of a set. For the purposes of this discussion a set will be defined as a collection of objects which have one or more characteristics in common. It is of course on the basis of this characteristic (or characteristics) that any set is different from other sets as a whole.

As an example of a familiar set, we might give that of the different coins whose values are equal to one dollar.

A FAMILIAR SET OF EQUIVALENT VALUES

1 dollar
2 half dollars
4 quarters
10 dimes
20 nickels
100 pennies

It has appealed to the writer to use this concept of sets of equivalent values to develop the basic concepts of fractions. He feels that the set concept is immediately applicable to the presentation of fractions at the fifth and sixth grade levels.

Perhaps the most basic principle of fractions is that of the set of values equivalent to the number 1. These are:

$$1=\frac{1}{1}=\frac{2}{2}=\frac{3}{3}=\frac{4}{4}=\frac{5}{5}=\frac{6}{6}=\frac{7}{7}=\cdots=\frac{n}{n}$$

If this basic principle is grasped thoroughly, the writer feels that other principles and processes of fractions will be more easily understood and developed without much difficulty. For example, we may then develop sets of equivalent values by using only a specified portion of the values listed in the above set in each case.

Thus:

$$\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}=\frac{5}{10}=\frac{6}{12}=\frac{7}{14}=\cdots=\frac{1\times n}{2\times n}$$

or

$$\frac{1}{3}=\frac{2}{6}=\frac{3}{9}=\frac{4}{12}=\frac{5}{15}=\frac{6}{18}=\frac{7}{21}=\cdots=\frac{1\times n}{3\times n}$$

also

$$\frac{2}{3}=\frac{4}{6}=\frac{6}{9}=\frac{8}{12}=\frac{10}{15}=\frac{12}{18}=\frac{14}{21}=\cdots=\frac{2\times n}{3\times n}$$

and similarly

$$\frac{3}{4}=\frac{6}{8}=\frac{9}{12}=\frac{12}{16}=\frac{15}{20}=\frac{18}{24}=\frac{21}{28}=\cdots=\frac{3\times n}{4\times n}$$

It may be pointed out that the numerators of these sets are either the natural numbers listed in order or are the successive values from some multiplication table. The denominators can be obtained in a similar manner. We are of course in each case subdividing the most elementary member of each set first into two parts, then three parts, and so on as needed to obtain successive members. As the number of parts into which the unit is divided increases, the number of parts needed to maintain the original value increases in proportion also.

After the concept of equivalent valued fractions has been developed and the method of obtaining successive values is understood, the application to the process of addition of fractions may be made immediately. Since fractions can be added only when their denominators are equal, only certain members of the sets of values will be immediately usable in each problem which we wish to work.

For the purpose of illustrating this principle, the writer developed a device which consists of a board 36″×27″ with grooved channels on it. Slides are placed into these channels on which are written the sets of equivalent values as developed above. These slides may be pushed to the left or right as necessary to "line up" the values which may then be added. For example, if we wish to add $\frac{1}{2}$ and $\frac{1}{3}$, the slides would appear as below.

$$\frac{1}{2}=\frac{2}{4}=\left|\frac{3}{6}\right|=\frac{4}{8}=\frac{5}{10}=\frac{6}{12}=\cdots=\frac{1\times n}{2\times n}$$

$$\frac{1}{3}=\left|\frac{2}{6}\right|=\frac{3}{9}=\frac{4}{12}=\frac{5}{15}=\frac{6}{18}=\cdots=\frac{1\times n}{3\times n}$$

$$\longrightarrow=\frac{5}{6}$$

Of course $\frac{1}{2}$ and $\frac{1}{3}$ may be added also by using other equivalent values from the above sets. for example

$$\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}=\frac{5}{10}=\left|\frac{6}{12}\right|=\frac{7}{14}=\cdots=\frac{1\times n}{2\times n}$$

$$\frac{1}{3}=\frac{2}{6}=\frac{3}{9}=\left|\frac{4}{12}\right|=\frac{5}{15}=\frac{6}{18}=\cdots=\frac{1\times n}{3\times n}$$

$$\longrightarrow=\frac{10}{12}\text{ or }\frac{5}{6}$$

Other possibilities would be exhibited, if the slides were extended in length so as to include additional values equivalent to the $\frac{1}{2}$ and $\frac{1}{3}$. In general the fractions $\frac{1}{2}$ and $\frac{1}{3}$ can be added whenever the members of each of the two sets involved have equal demoninators.

Similarly $\frac{2}{3}$ and $\frac{3}{4}$ may be added as below.

$$\boxed{\frac{2}{3}=\frac{4}{6}=\frac{6}{9}=\left|\frac{8}{12}\right|=\frac{10}{15}=\frac{12}{18}=\cdots=\frac{2\times n}{3\times n}}$$

$$\boxed{\frac{3}{4}=\frac{6}{8}=\left|\frac{9}{12}\right|=\frac{12}{16}=\frac{15}{20}=\frac{18}{24}=\cdots=\frac{3\times n}{4\times n}}$$

$$\longrightarrow\frac{17}{12}=1\frac{5}{12}$$

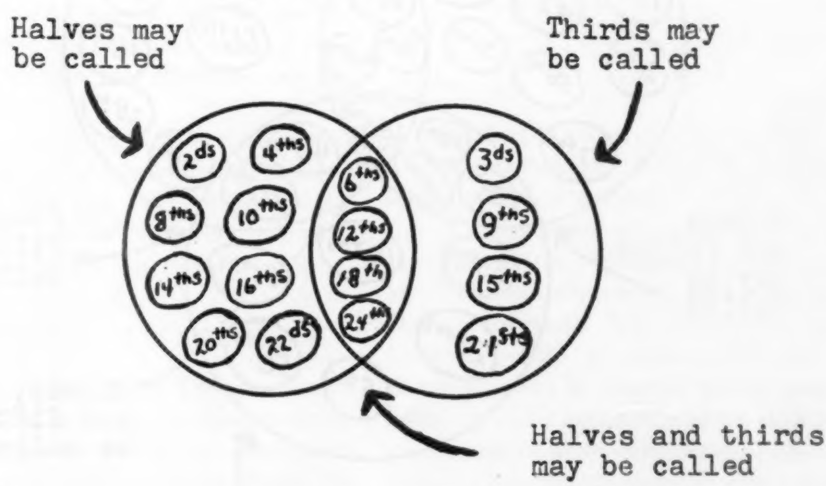
As a further means of illustrating the use of set concepts to develop the principle of common denominators, we may use the concept of the intersection of sets of values. For this purpose, diagrams were developed to show the members common to two or more sets. These diagrams show the different common denominators which may be used in adding two or more fractions. Since fractions can be added only when their denominators are the same, the numbers included in the interesections represent "possibilites" for accomplishing these additions.

Three different situations which may arise are shown in the following diagrams.

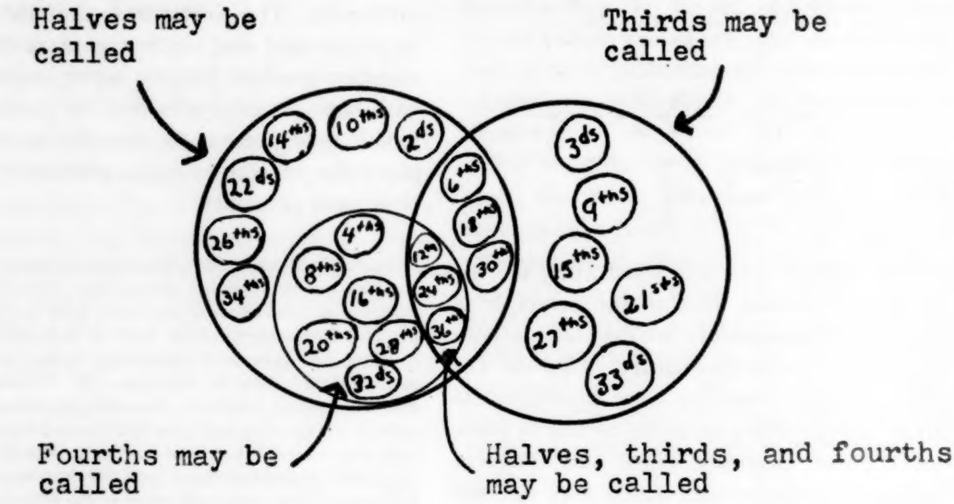
Many other applications involving the use of sets can undoubtedly be found in arithmetic. The writer feels that these will be discovered and made use of as the elementary teachers become better acquainted with basic set principles and the possibilities which they present for simplifying the explanation of certain basic arithmetic principles and processes.

EDITOR'S NOTE. "Set" language is slowly creeping into our professional discussions. This need not frighten anyone because the basic idea is very simple and is not particularly new. It is a useful concept for grouping and classifying items that have some characteristic in common. Dr. Hannon uses the overlapping circles to illustrate graphically how several sets overlap and how this shows certain common values and features of fractions. Teachers may stimulate resourcefulness and understanding by asking pupils to make sets such as the following: (a) expressions representing the value of one-half cent; (b) the set of rectilinear plane figures; and (c) a set of common fractions having the value one half. A pupil may ask, "Could the fraction 3½/7 be used as member of the "½ set"? And in reverse one might write the series: 2.5 ft., 2½ bu., 250%, and 5/2 and ask for various set identifications. At the college level the idea of set may become much more sophisticated.

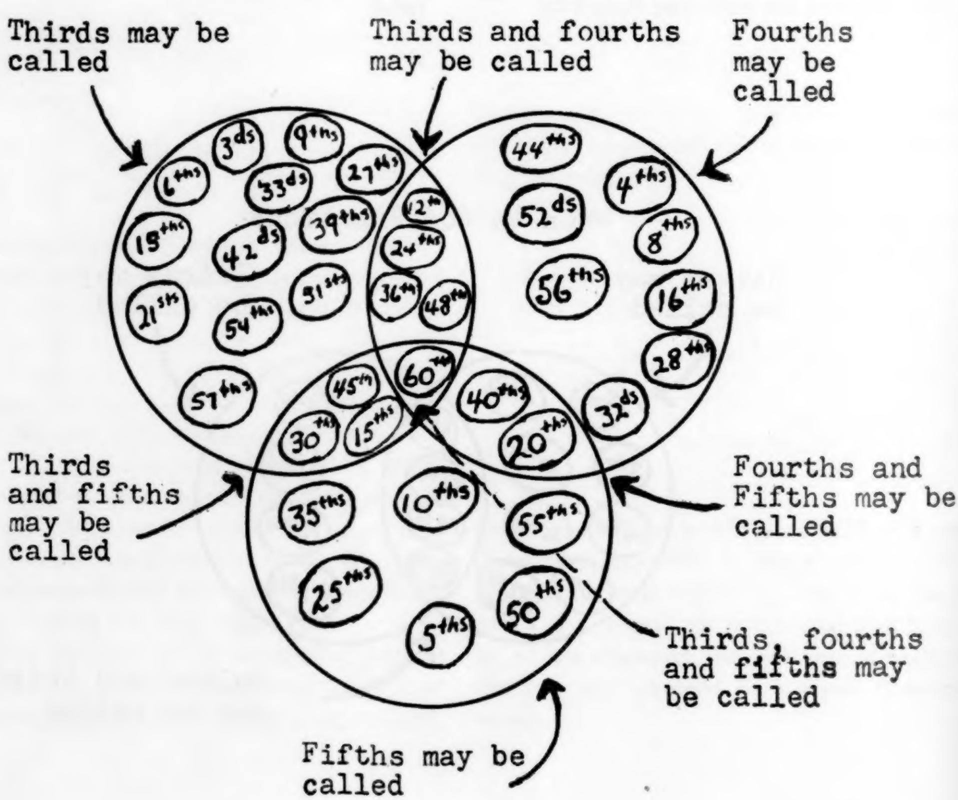
1. When We Add Fractions



2\. When We Add Fractions



3\. When We Add Fractions



"But the Teacher Didn't Show Us that Way"

John A. Sestanovich
Bryan School, Cresskill, N.J.

When did you hear these words last? You are a parent and still have a child of elementary school age; the last time you heard these words was probably last evening when you tried to help with the arithmetic homework. These words are doubtlessly heard in millions of homes every evening. Is there a reason for it? Is the teacher making it easier for himself? Or is he making it easier for the children, because "he didn't show us that way?" Let's see if we can find out.

I like to use an illustration that clearly points out the crux of the problem concerned. The situation is at a lower level, but I feel the principle involved applies elsewhere also.

$$\begin{array}{r} 34 \\ -19 \\ \hline 15 \end{array}$$

The example above is in subtraction. How would you solve it? I trust that everyone reading this who is over eight years of age will get the correct answer, but I'm not interested in that at this point. What I particularly want to know is *HOW* did you get the answer of 15?

In order to better understand the question before us, let's examine our arithmetic background. *How* you got your answer depends largely upon *how* you were taught.

Basically, there are two forms of subtraction:

1. ADDITIVE 2. TAKE-AWAY

Simple subtraction consists of examples such as the following, in which the number above is larger than the number in the corresponding colum below, or as we say in school, the number in the minuend is larger than the corresponding number in the subtrahend.

$$\begin{array}{r} 18 \\ -\ 6 \\ \hline \end{array} \qquad \begin{array}{r} 9 \\ -2 \\ \hline \end{array} \qquad \begin{array}{r} 3 \\ -1 \\ \hline \end{array} \qquad \begin{array}{r} 281 \\ -160 \\ \hline \end{array}$$

The *additive form* evokes a response of "six and what makes 18?"

The *take-away form* stimulates us to ask "six from 18 is what?"

No matter which word you use—from, minus, less, take-away—it's all the *take-away form* of simple subtraction. All the words imply taking away, which subtraction is. The latter form is the better of the two. It can be easily seen and be easily understood by the children. It has reasoning! It has meaning! "Six from 18 is what?" That is subtraction. "Six and what make 18?" is something else. The basic thought response is incorrect. Personally, I feel this additive form has no place in our modern education. We'll speak no more of it. It is not our basic concern at the moment.

Let's get back to our original example.

$$\begin{array}{r} 34 \\ -19 \\ \hline 15 \end{array}$$

The example above is called a compound subtraction example. Primarily, it is not a simple subtraction example because it consists of digits above which are smaller than the digits in the corresponding column below. This instantly creates chaos in the habits of any third grader. When he sits down to his homework—it's a calamity—hands are thrust skyward in despair and groans are heard for miles around. It seems that death itself would be more welcome than, above all things, a subtraction example with a smaller digit on top. The teacher pulled a sneak—it's a dirty trick! This is not a laughing

matter to a serious-minded third grader—to him his whole world has just collapsed with the advent of this compound subtraction example. "Subtraction is one thing—but this is too much!"

A few minutes later, when our third grader dries his eyes and makes Mommy promise to get Daddy to help, a little calm is restored to a once peaceful household. And so after the dinner dishes are done and Dad has his cigar, he finally opens the book and embarks on the adventure of his career.

$$\begin{array}{r} 34 \\ -19 \\ \hline \end{array}$$

"First you look at it, son, the nine can't go into the 4 so we have to borrow one from the 3. Now we say 9 from 14 is 5. We put the 5 down below in the answer column. We borrowed one so the one becomes a two. Two from three makes one. The answer is 15. Simple, wasn't it?"

At this point the most cruel thing in the world is about to take place. Dad's victorious smile is going to be wiped from his beaming face with the most immortal words of the ages, "But the Teacher didn't show us that way!" whereby Dad exclains, "You mean there's another way? Gee, I must be getting old!"

Well, Dad, I guess you are getting old—and there is another way—a better way—a more meaningful way to subtract.

In compound subtraction we use (1) Decomposition or (2) Equal-Additions methods. Of course, the additive method also applies but we're not even considering it.

The Equal-Additions method which Dad showed to Johnny is all mechanical. There is no meaning or logic involved. All Dad knows (and it's not his fault, because that's the way he was taught) is that be borrowed and he has to make up for it. Where is a more conveneint place to make up for it than at the bottom of the next column to the left? Is it surprising that he came up with the right answer? No, it's not surprising at all. It makes no difference whether you subtract 2 from 3 or 1 from 2—you still get one.

You"ll get the right answer all the time but actually you don't realize the meaning behind the method. You don't realize what you are doing to get your correct answer. How under the sun can anyone explain to a third grader, or justify the fact, that he borrows from one number and changes another number because of it? I don't think you can explain, let alone justify.

If all you want for your child to get out of arithmetic is the right answer, then your problem is solved. Teach him the Equal-Additions method. It might take awhile, especially if he's a child who wants to know why, but he'll get the right answers with it just as Dad does. But, if you'd like Johnny to understand the *HOW* and the *WHY* of the beauties of arithmetic—and it is beautiful because of its logic and meaning, then read the rest of this article and think awhile!

The method which Johnny's teacher showed him—at least I hope so—is called the Decomposition method. The response evoked is something like this—

$$\begin{array}{r} 34 \\ -19 \\ \hline 15 \end{array} \quad \text{becomes} \quad \begin{array}{r} 2 \\ -1 \\ \hline 1 \end{array} \quad \begin{array}{r} 14 \\ -\ 9 \\ \hline 5 \end{array}$$

"I can't take 9 from 4 so I must borrow. I take 1 ten from 3 tens to enable me to make 14 ones. 9 from 14 is 5. Place the 5 in the answer column. I borrowed 1 ten from 3 tens so it now becomes 2 tens. One ten from 2 tens is 1 ten. Answer 15. Briefly, 9 from 14 is 5. 1 from 2 is 1. 15. A little more meaningful, don't you think?

Here the child realizes that since he borrowed, the number that he borrowed from is the one that gets smaller and not the other. The Decomposition method can easily be objectified for the children as follows:

$$\begin{array}{l} \text{xxx 1111} \\ \text{x 111111111} \\ \hline \\ \end{array} \quad = \quad \begin{array}{l} \text{xx 11111111111111} \\ \text{x 111111111} \\ \hline \text{x 11111} \end{array}$$

3 tens and 4 ones is the same as 2 tens and 14 ones.

The Equal-Addition method would be hopeless to explain in this way—it certainly couldn't show *WHY*.

I'm sure that if you were taught as Johnny's Dad, you are not less efficient than a person who uses the Decomposition method. That's not the question. If fact, you may even get your answer faster—if that's what you want. But if we want meaning put into our arithmetic at the elementary level in order to insure a future free of "arithmetic illiterates"—people who know the mechanics but not *WHY?*, then we need more meaningful arithmetic in our schools. The trend is toward more meaning—let's hope it continues and when it's explained more fully, more Dads and Moms will realize the reasons and advantages of why "The Teacher Didn't Show Us That Way?"

Say, Dad, 19 from 34 is 15. How did you say you got that answer?

EDITOR'S NOTE. The editor believes that each of the methods commonly used for subtraction can be rationally explained so that children can understand. Further, he does not like the term "borrow" used with the Decomposition method. He prefers to explain this method in the sense of "changing" the structure of the number in the minuend just as when one must pay out $3.45 and has only a five-dollar bill. Some "changing" must be made in the structure of the five dollars before the amount of $3.45 can be paid.

As long as we keep changing methods of work with numbers from one generation to another we will have some conflict between home and school. This comes even though we have no homework. These conflicts are easily resolved in a school that has close ties of parents and teachers. Some schools try to avoid conflicts in learning by restricting home assignments to types of work where different modes of thinking and work may be an asset. Other schools permit different pupils to work by different methods. At what grade-level should homework we started? Does homework really pay dividends in the elementary school? Do the factors of interference by parents seriously affect the values in homework? How can a school capitalize upon the contributions that may be made by a parent group that is perhaps better educated than the teachers and school officials?

Rounding Numbers

WILBUR HIBBARD
Highland Park, N. J.

IN THE NORMAL SEQUENCE of arithmetic we study whole numbers, common fractions and decimal fractions in this order. After pupils have gained some facility with decimals, we direct them to round their answers to the nearest thousandth or other decimal. This rounding is difficult for two reasons. First, the meaning of the term nearest thousandth and in the second place we have the mathematical immaturity of the pupil.

We have taught them that numbers are fixed, definite and tamper proof. Now we say "round to the nearest hundreth." Frequently, pupils ask, "Throwing this away?", "What happens to the remainder?", "What happens if I come to thousandths before all the numbers are used up?"

Most 7th graders know the meaning of the word nearest. They can name the nearest city, river, person, etc. They know it is 8 miles to *A* and 12 miles to *B*. So it is nearer to *A* than *B*. This is real. But, who has ever seen a thousandth? So how can one determine the nearest thousandth?

Perhaps this approach would help. If we are asked to round a number to the nearest hundred, the answer will be of this form—100, 200, 300, $\cdots$ 1100, 1200, 1300. For example, round 1236 to the nearest hundred. To the nearest hundred means that our answer will be the result of counting by hundreds and will be one of the above sequence numbers.

The number 1236 is more than 1200 and less than 1300. It is between these two hundreds. Which is nearer? It is 36 points from 1200 and 64 points from 1300. Therefore, 1236 is rounded to 1200, the nearest hun-

dred. "Do I throw away the 36?" Actually no. We are grouping the numbers around hundreds, the nearest hundred. Hence 1236 is placed with 1200, the nearest. Our number is still 1236.

Do we throw anything away or add something if we direct a pupil to take the nearest vacant seat, or to go to the nearest blackboard?

Similarly round 1263 to the nearest hundred. It is between 1200 and 1300. It is 63 points from 1200 and 37 points from 1300. So it is nearer 1300 and is so placed. A number scale is useful in showing the relative positions.

It might be well to develop a rule. Example, 1236 to the nearest hundred. Cover all digits to the right of the hundreds position. We see 1200 plus something covered. Is this amount a half hundred? More, or less? Since 36 is less than half a hundred, it is nearer 1200. Similiarly locate 1225, 1245, 1258, etc., noting the position on the scale. Use several numbers on both sides of 1250. This shows that all numbers having 1, 2, 3, 4, in the tens place are nearer 1200, while all numbers having 5, 6, 7, 8, 9, in the tens place are nearer 1300. For uniformity we place the exact midpoint 1250 with the larger hundred. Now, the rule may be stated. In rounding to the nearest hundred, cover all digits to the right of hundreds. If the digit to the right of hundreds is 5 or larger, take the larger hundred; if that digit is 4 or less, take the smaller hundred.

1200

1250

1263

1300

Thus the rule is established. The rule is extended to cover hundredths, etc. Check the digit to the right of the decimal position required.

Round 14.3792473—to the nearest hundredth. Counting by hundredths we get this sequence—.01, .02,—14.36, 14.37, 14.38 Our number is between 14.37 and 14.38. Check the digit to the right of hundredths place. It is 9. Using the rule mentioned earlier, we find our number is nearer 14.38 than it is 14.37.

Round the same number to the nearest thousandth. It is more than 14.379 but less than 14.380. Check the digit to the right of thousandths. It is 2. Using the rule, we say our number is nearer 14.379.

Rounding numbers might be compared withfiling numbers in a cabinet. If we have the number 14.3792473 (heaven forbid) on a card and want to file it in a case scaled to hundredths, just where shall we put it? The nearest location would be 14.38. We do not change the numbers on the card. We file it.

Does this approach help answer the opening questions? Do I throw the remainder away? We disregard it. How many zeros do I add? Enough to take us to hundreths, or thousandths and to the next digit on the right.

Do I have to add zeros? Perhaps. Can you write the answer required? If not, add as many zeros as needed.

What happens if I reach thousandths before all the numbers are used up? Carry the answer to one place to the right of thousandths and stop. You have met the requirements of the problem. No reason to continue.

An extension of rounding numbers might include rounding to the nearest 5, 12, 25, or 50.

EDITOR'S NOTE. The principles we use in "rounding" numbers assume an understanding of the decimal number system and with this base rounding should not be difficult to understand. We use "rounded" numbers in estimating and in expressing approximations. For many purposes we can obtain a useful answer by using simple approximations. For example, in many cases the approximation of $\pi=3$ serves as well as $\pi=3.14$ or $\pi=31/7$ and is a good deal easier to use.

In certain computations, as for example in statistical work, we encounter a need for good judgment as to when a number should be rounded and the extent to which it should be rounded in order to obtain an answer several computational steps later and one that is perhaps correct to the nearest tenth. e.g. statistical measures such as standard deviation, critical ratio, and coefficient of correlation. Likewise we face the error that accumulates through consistently rounding a digit 5 to the next higher value. Also, the prevailing practice in most markets to "round up" any fraction of a cent to the next higher cent seems to violate our basic principle of rounding. Rounding is a fine topic for developing thinking and understanding. It must also be accompanied with some information about *how* and *when* in relation to certain uses.

Help in Problem Solving

CATHERINE GEARY
Middletown, Connecticut

WHEN CHILDREN DO PROBLEMS mentally (figuring them out without writing them down), it is well to ask them to tell what they thought in order to get the answer. This shows that problems may be done in different ways, it helps other children to understand difficult problems, and talking about a problem helps to fix a method of doing a certain kind of problem in the mind of the person who discusses it. It is well to remember that a child may tell the correct answer and yet may write it incorrectly. For instance, in the problem, "If one pencil costs 5¢ how much would 6 pencils cost?", the child may answer correctly, 30¢. However, if he is asked to write it, he may omit the cents' sign (¢). In that case the problem would be considered incorrect. For this reason, writing problems on the board helps the child to see that sometimes a word or a sign must be included in order to have a totally correct answer.

When problem tests are given in school no help is given to the children. They learn more if they can figure out the answers for themselves. After the test has been corrected, the class works the more difficult problems on the board.

The following points should aid a person helping a child to do written work in problem solving.

1. Read the problem carefully. Ask yourself, what does the problem ask me to find? Then decide whether you should add, subtract, multiply, or divide in order to get the answer.

2. Before you do any of these things on paper, *estimate* your answer. That means you should figure mentally *about* what your answer should be.

Problem: "If you were buying one game that cost $.49 and another game that cost $.98, how much money would you spend for both?"

In estimating your answer you would think—$.49 is *about* $.50 and $.98 is *about* $1.00. *One dollar* and *fifty cents* is $1.50, so both games would cost *about* $1.50.

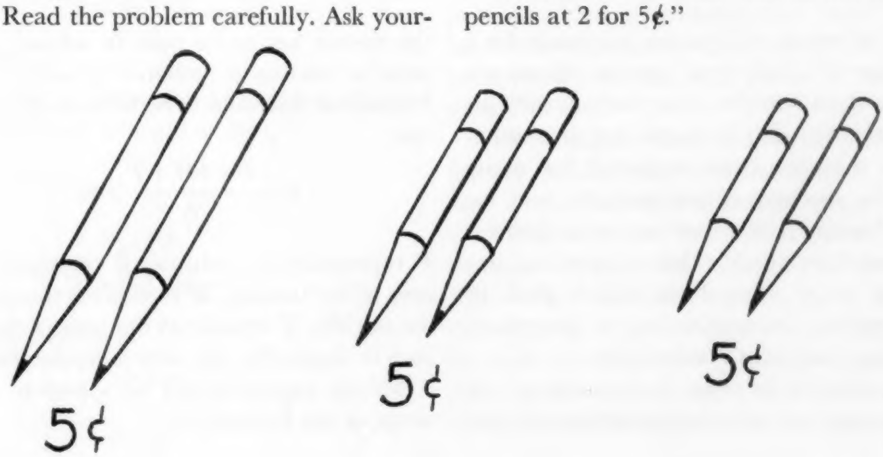
3. Do the problem. See if your answer is about the same as what you estimated it would be. If the *estimated* answer and the *real* answer are about the same, the problem is probably correct.

4. If you are unable to estimate an answer because you do not know whether to add, subtract, multiply or divide the following suggestions are helpful.

a. Use smaller numbers than those stated in the problem, this will help you to decide how to do the problem.

b. Make a picture or drawing, so that you can *see* the answer.

Problem: "Find the cost of six colored pencils at 2 for 5¢."



From a drawing like the one above, a child can see that 6 pencils cost 15 cents.

5. Be sure to answer every question asked in a problem.

6. Add, subtract, multiply, and divide correctly.

7. Write words and figures carefully and clearly.

Finally, the following general suggestions should aid the parent in helping the child solve problems.

1. Allow the child to solve problems mentally, and to tell what he thought in getting his answer.

2. Work with the children in solving difficult problems. Sometimes just asking the child a question will guide him to correct thinking. A good question is better than having a discouraged child.

3. Give problem tests in which the child has to discover answers for himself. (He can not have someone to guide him at all times.)

4. Go over corrected tests and try to have the child discover the mistakes he has made.

5. Remember that the child learns to solve problems by doing just that—solving problems.

The Prismoidal Formula

G. T. BUCKLAND
Appalachian State College, Boone, N. C.

JUST AS THERE IS a master key made for a series of locks, and just as there are usually short cuts for doing various difficult tasks, there is also a master key in mathematics that has been neglected for many years by teachers of mathematics and for many hundreds of others was never known. The simplicity of a key that is capable of unlocking many other locks which each in turn requires a special key is sometimes amazing, truly so in mathematics.

A wonderful formula, in fact one of the most important formulas in arithmetic and the master key to be used in solving problems in relation to volumes of solids is the Prismoidal Formula. The formula is simply this:

$$V=\frac{B+4M+T}{6}\times H$$

V represents the volume, B represents the area of the bottom, M represents the area of the middle, T represents the area of the top and H stands for the perpendicular height.

Several examples will be solved to show usage of the formula.

EXAMPLE 1

Find the volume of a rectangular solid 4 ft. long, 3 ft. wide and 2 ft. high.

Common Formula	*Prismoidal Formula*
$V=lhw$	$V=\frac{B+4M+T}{6}\times H$
$V=24$ cu. ft.	$B=12$ (sq. ft.)
	$4M=48$ (sq. ft.)
	$T=12$ (sq. ft.)
	$H=2$ (ft.)
	$V=24$ cu. ft.

EXAMPLE 2

Find the volume of a cylinder having a diameter of 10 ft. and a height of 12 ft.

Common Formula

$$V=\pi r^2 h$$

$$V=942.48 \text{ cu. ft.}$$

Prismoidal Formula

$$V=\frac{B+4M+T}{6}\times H$$

$$B=78.54 \text{ (sq. ft.)}$$

$$4M=314.16 \text{ (sq. ft.)}$$

$$T=78.54 \text{ (sq. ft.)}$$

$$H=12 \text{ (ft.)}$$

Substituting:

$$V=942.48 \text{ cu. ft.}$$

EXAMPLE 3

Find the volume of a cone having a diameter of 12 ft. and a height of 20 ft.

Common Formula

$$V=\tfrac{1}{3}\pi r^2 h$$

$$V=753.984 \text{ cu. ft.}$$

Prismoidal Formula

$$V=\frac{B+4M+T}{6}\times H$$

Therefore:

$$B=113.0976 \text{ (sq. ft.)}$$

$$4M=113.0976 \text{ (sq. ft.)}$$

$$T=0$$

$$H=20 \text{(ft.)}$$

$$V=753.984 \text{ cu. ft.}$$

EXAMPLE 4

Find the volume of a sphere having a diameter of 10 ft.

Common Formula

$$V=\frac{4}{3}\pi r^3 \text{ or } \frac{4\pi r^3}{3}$$

$$V=523.6 \text{ cu. ft.}$$

Prismoidal Formula

$$V=\frac{B+4M+T}{6}\times H$$

$$B=0$$

$$4M=314.16 \text{ (sq. ft.)}$$

$$T=0$$

$$H=10 \text{ (ft.)}$$

$$V=523.6 \text{ cu. ft.}$$

The Prismoidal Formula is a jewel. One formula is easy to remember while many might be forgotten. This is only one of many examples of amazing mathematical facts, formulas, and procedures that men even have spent life times in developing but have been placed on the shelf by other men to gather dust.

This formula is applicable to all phases of finding volume provided the form is regular. As you know, in the olden days, when gold was mined in the western states a lot of waste ore was thrown into huge dumps. Present day methods have found a lot of rich ore in these heaps. The Prismoidal Formula represents one huge glitter of gold that has been resurrected from the mathematical accumulations of times long past.

The 37th Annual Meeting of the National Council of Teachers of Mathematics

Dallas, March 31–April 4

"THE EYES OF TEXAS" will indeed be focussed on the National Council of Teachers of Mathematics from March 31 through April 4, 1959 when the 37th Annual Meeting of the Council will be held in the city of Dallas located in the Lone Star State. While headquarters for the convention will be in the Baker Hotel, sectional meetings are scheduled in both the Baker Hotel and the Adolphus Hotel just across the street.

To provide a genuine introduction to the spirit of the southwest a "barbecue dinner," with entertainment, has been planned for Wednesday evening, April 1, and a cordial invitation is extended to all who plan to attend the convention. Come early and enjoy this ranch style meal, scheduled to be held in the Texas State Fair Park, the home of the world's largest state fair and of the Cotton Bowl.

The Twenty-fourth Yearbook

One of the highlights of the program will be the presentation of the Twenty-fourth Yearbook, *The Growth of Mathematical Ideas.* This volume, prepared under the editorial direction of Phillip S. Jones, promises to be another of the many outstanding contributions of the Council to the professional literature related to mathematics education. Under the general heading, "The Growth of Mathematical Ideas," five sessions, beginning Thursday afternoon and continuing through Saturday afternoon, have been arranged to consider the themes and concepts included in the Yearbook. Those who believe that mathematics is more than a disconnected sequence of subjects, such as arithmetic, algebra, and geometry, will be pleased with this emphasis on the continuity of mathematical concepts. Early in his experiences the child is introduced to such ideas as number and operation, relation and function, measurement and approximation, probability and statistics, proof and symbolism, and to nourish the healthy growth of these ideas from the early years in the elementary school through the later years of the secondary school and beyond is the responsibility of the mathematics teacher. These and other concepts considered in the Yearbook provide the kind of continuity which enriches the study of mathematics and gives it meaning and significance. Each of these five sessions will be led by the authors of the sections under consideration and will include a panel discussion by qualified teachers from elementary, junior and senior high schools.

General Sessions

At the first of the three general sessions E. G. Begle of Yale University will discuss the program of the School Mathematics Study Group, the work that has already been done and plans for the future. Recognizing that this and other curricular studies have long range implications for the education of mathematics teachers, Kenneth E. Brown, Mathematics Specialist of the U. S. Office of Education, has made a comprehensive study of the background and preparation of those who are teaching mathematics now. He will present his findings at the second general session under the title "The Qualifications and Teaching Load of Mathematics Teachers" while at the third general session Robert Fisher of The Ohio State University will discuss "The Development of the Mathematics Teacher for Tomorrow." The Saturday luncheon speaker is John W. McFarland, superintendent of Schools, Houston, Texas, and he has selected "A Time of Opportunity for Mathematics Teachers" as his topic. L. D. Haskew, Vice President of the University of Texas, has often

been referred to as "the finest speaker that ever came out of Texas" and if he can make that kind of a record *outside* the state it is not difficult to imagine what he will do when he speaks *inside* the state at the banquet on Friday evening. His theme, "The Higher Education," may not mean much to you now but it will have a much richer significance once you have listened to his interpretation of these three words.

Sectional Meetings

Included in the program are approximately thirty-five sections planned to meet the varying needs and interests of all who attend the convention. Problems in the teaching of mathematics from the early grades of the elementary school through undergraduate work on the college level are considered in special sections under the direction of able and competent leaders in the field of mathematics education. Illustrative of such problems are "Geometry in the Elementary School Curriculum," "Developing Algebraic Concepts in Grades 7 and 8," "Coordinate Geometry in the Plane Geometry Course—What and How?" and "Concept Learning in Differential Calculus." The interest of teachers in "modern" mathematics is reflected throughout the program and sections are included to consider such topics as "Number Pairs in the Elementary School," "The Impact of Modern Mathematics on the Mathematics Curriculum of the Seventh and Eighth Grades," "Presentation of Specific Topics in Elementary Algebra Using Modern Language" and "Modern Introductory Mathematics for College Freshmen."

Curriculum trends on all levels will be discussed by qualified speakers, experimental studies now in progress will be reported and their implications considered. Other special sections deal with a variety of topics, such as "Research in Mathematics Education," "The Role of Evaluation," "Cooperation with Industry," "Teaching by Television," "Mathematical Contests," "Teacher Education" and our relations with the teachers of mathematics in other countries.

Nor has the program neglected the large and increasing interest in the important problem of providing for the academically talented student. Consistent with this interest the Student Forum is concerned with "Calculus for the Tenth Grade" and, in addition, sectional meetings are devoted to the discussion of "Programs of Acceleration" and "A Summer program for Talented High School Students" along with such specific topics as "Enriched Material for Superior Students in High School Algebra" and "Freewheeling with the Gifted."

Three laboratory sessions are available, one for elementary school teachers, one for junior high and one for senior high school teachers. In each of these laboratories selected teaching aids will be developed and each participant will actually construct teaching devices which will be helpful in his own classroom. School exhibits of student projects will be stimulating and suggestive, while commercial exhibits will include the latest textbooks along with a wide variety of teaching equipment. Provision will also be made for those who are interested in previewing many of the latest films designed to improve the teaching of mathematics.

Of special interest to those fortunate persons who preregister for them, will be the two lecture-demonstrations of the Remington-Rand Scientific Computer at Southern Methodist University. This is one of the largest installations on any university campus, and this opportunity should prove of great value to those who wish to know more about "Univac." Opportunities for visits to Dallas Schools and tours to other places of interest are also being arranged.

To be present at this convention is to enjoy the finest professional experience available to mathematics teachers. More than one hundred persons have program responsibilities. All of them are highly successful teachers and many of them are distinguished leaders in the

field of mathematics education. More general and sectional meetings than at any previous convention have been planned. Texas is a state that refuses to think small and there are rumors abroad that the mathematics teachers of the Lone Star State are expecting to break the attendance record established at Cleveland in 1958. Not only are "the eyes of Texas" watching but so are the eyes of all those who have a deep and genuine interest in improving the quality of teaching in the mathematics classrooms of America.

Arithmetic Programs

Sectional meetings dealing with arithmetic in the elementary school will be spread throughout the conference. Topics and speakers will include: "Blocks to Arithmetic Understanding"—C. C. Collier of Michigan; "Allotment of Classroom Time to Arithmetic in the Middle Grades"—Donald E. Shopp of Louisiana; "Needed Research in Arithmetic"—E. Glenadine Gibb of Iowa; "The Insatiable Quest: Mathematicking"—Marguerite Brydegaard of California; "Geometry in the Elementary School Curriculum"—Patrick Suppes of California; "Mathematics for the Slow Learner"—Charlotte Junge of Michigan; "Related Basic Concepts as Extras for Gifted Children"—Jesse Osborn of Missouri; "An Investigation in Providing for Individual Differences in Arithmetic"—Frances Flourney of Texas; "Generalizations in Arithmetic Learning"—John R. Clark of Pennsylvania; and "Number Pairs in the Elementary School"—H. Van Engen of Wisconsin.

The discussions following the presentations together with informal exchange of ideas at these conferences add interest and value for the people who attend.

"Today my heart beat 101,368 times; my blood traveled 168,000,000 miles; I breathed 22,839 times; I inhaled 430 cubic feet of air, and exercised 7,000,000 brain cells . . . so I'm too tired to do homework."

Reprinted from The Chicago Tribune

R_x Ratio

Genevieve Forrest
East Central School, Irondequoit, N.Y.

May I share with you a project which furthers an understanding of ratio, introduces proportion, and serves as another illustration of similarity?

Several days before you expect to begin the project, ask the children to bring to school small simple pictures about 2 by 3 inches in size. Warn that too much detail will be difficult. Note paper, Christmas cards and younger children's coloring books are good sources of material. Needless to say, the teacher should have a supply for the usual forgetful few.

Begin by distributing 9×12 drawing paper or graph paper. The paper is then squared off into one-inch squares. The lines should be drawn lightly, otherwise the lines, not the picture, stand out.

Once this is done each one must decide on the ratio he will use. The pupil measures roughly how many times longer and how may times wider the paper is than his picture. If the paper is four times larger, he decides the ratio should be 1:4. Some will use 1:2 and others even a 1:1 ratio.

Once the ratio has been established, the picture is squared off too. For example, if a 1:4 ratio is to be used, then the picture must be squared off in one-fourth inch squares. Next, letter each block of the picture across the top and number each block down the side. Then, letter and number the squared off paper correspondingly.

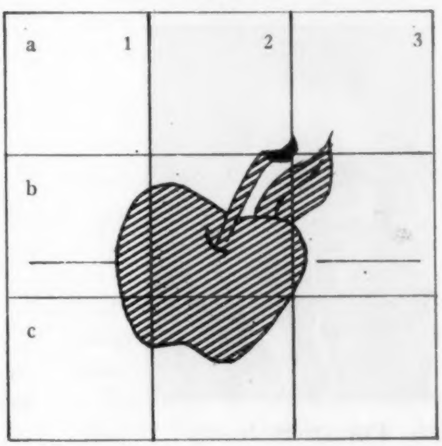
Before beginning the project, the teacher should have on the blackboard one area of one inch squares and a second area of four inch squares lettered and numbered as described above. A picture should be sketched on the smaller area. With this he can demonstrate how to copy the picture block by block.

Once the picture is copied, it should be colored in its original colors, mounted on colored paper, and the ratio used stated.

Watch the look of pride and satisfaction on each child's face as he discovers his picture after the work has been posted. You are sure many are saying, "Is this mathematics? I can do this." Many of the less gifted have a real sense of accomplishment. Often this is followed by a definite improvement in attitude and subsequently in daily work.

An evaluation of the resulting variety of pictures offers innumerable opportunities to further an understanding of ratio, proportion and similarity.

Once you try this project, I believe you will agree that it is rare good fun and a rewarding experience.



NCTM

THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Report of the Nominating Committee

The Committee on Nominations and Elections presents its slate of nominees for offices to be filled in the 1959 election. The term of office for the two vice-presidents is two years. Three directors are to be elected for terms of three years.

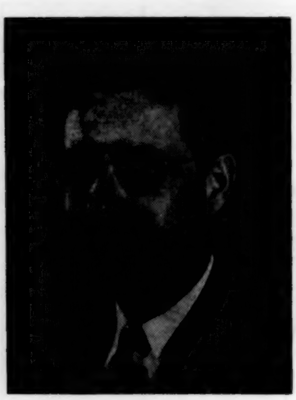
In making nominations for the three director positions, the Committee followed the directive adopted by the Board of Directors in 1955 which states, "Nominations shall be made so that there shall be not more than one director elected from each state, and that there shall be one director, and not more than two, elected from each region." Members may consult THE MATHEMATICS TEACHER for October, 1955, for a map of the regions as they are now defined.

Ballots will be mailed on or before February 10, 1959 from the Washington Office to members of record as of that date. Ballots returned and postmarked not later than March 10, 1959 will be counted.

The Committee wishes to thank the many members of the NCTM for help in giving their suggestions for nominees. It is hoped that all members of our organization will be sure to exercise their privilege of voting.

MILTON BECKMANN, *Chairman*
CLIFFORD BELL
CHARLES BUTLER
ROBERT FOUCH
MARTHA HILDEBRANDT
MILDRED KEIFFER
ANN PETERS
MYRON ROSSKOPF
MARIE WILCOX

NOMINEES FOR VICE-PRESIDENT—COLLEGE LEVEL



PHILLIP S. JONES

Z. L. LOFLIN

Phillip S. Jones

Professor of Mathematics and Education, University of Michigan, Ann Arbor, Michigan.

A.B., Ph.D., University of Michigan.

Jackson Junior High School, Jackson, Michigan; Edison Institute of Technology, Dearborn, Michigan; University School, Ohio State University; Duke University.

Member: NCTM; AMS; MAA; Phi Rho Pi; Sigma Xi; Phi Kappa Phi; and AAAS.

Activities in NCTM: Member, Board of Directors; Departmental Editor, THE MATHEMATICS TEACHER; Editor, *Twenty-fourth Yearbook;* former Associate Editor, THE MATHEMATICS TEACHER.

Other activities: Member, NAS-NRC Committee; SMSG Advisory Committee; Mathematics Committee, C.E.E.B.; formerly member of Board of Governors of MAA.

Publications: Numerous articles in THE MATHEMATICS TEACHER, *The Monthly*, and other magazines.

Z. L. Loflin

Professor and Head Mathematics Department, Southwestern Louisiana Institute.

B. S., M. S., Louisiana State University; Ph.D., Columbia University.

Member: NCTM; Phi Kappa Phi; Pi Mu Epsilon; Kappa Mu Epsilon; Louisiana Teachers Association; NEA; American Educational Research Association; American Mathematical Society; American Chemical Society; AAUP; American Institute of Mining and Metallurgical Engineers; American Association for Engineering Education; Central Association of Science and Mathematics Teachers; Rotary Club of Lafayette. Listed in *American Men of Science* and *Who's Who in the South and Southwest.*

Activities in NCTM: Editorial Board, THE MATHEMATICS TEACHER.

Other activities: Member, National Board of Governors of Mathematical Association of America; Past National President, Theta Xi.

NOMINEES FOR VICE-PRESIDENT—JUNIOR HIGH SCHOOL LEVEL

MARIAN C. CLIFFE

MILDRED B. COLE

Marian C. Cliffe

Supervisor of Mathematics Curriculum, Junior and Senior High Schools, Los Angeles City Board of Education.

B. A., University of California at Los Angeles; M.A., Ed.D., University of Southern California.

Teacher of mathematics, junior and senior high schools, Los Angeles city;

evening instructor, Los Angeles City College; chairman mathematics department, Verdugo Hills Six-Year High School; junior high school counselor, Verdugo Hills High School; co-ordinator, director of summer workshops for teachers of mathematics, Los Angeles city schools; consultant for mathematics workshops, institute speaker in various districts.

Member: NCTM; California Mathematics Council; Council of Directors and Supervisors; American Association for the Advancement of Science; Texas Council of Teachers of Mathematics; Los Angeles City Teachers Mathematics Association; Society of Delta Epsilon; Pi Mu Epsilon; Pi Lambda Theta; Delta Kappa Gamma.

Activities in NCTM: Membership Committee, 1955——; Committee on Secondary School Standards, 1956–58; Cochairman of Local Arrangements, Sixteenth Summer meeting, 1956; Treasurer, Christmas meeting, 1953; on NCTM convention program at Los Angeles, Boston, Milwaukee, and Indiana University.

Other activities: State President, California Mathematics Council, 1957–59; President, Southern Section, CMC, 1955–57; Treasurer, Southern Section, CMC, 1953–55; Member, Board of Directors, CMC, 1951–53; Chairman, Subcommittee on Secondary Mathematics for Non-College Preparatory Students, California Subcommittee on Content and Sequence of Mathematics from the Ninth Through Fourteenth Grades; Delegate, Industry-Education Conference, 1957 and 1958; Secretary, Society of Delta Epsilon (Doctoral Society USC), 1958–59; Treasurer, Delta Epsilon, 1957–58; Faculty President, Verdugo Hills High School, 1951–52; Planning Committee, California Conference for Teachers of Mathematics, 1953–58; California State Scholarship Committee, Delta Kappa Gamma, 1957–59; Chairman, Curriculum Branch Staff, Los Angeles City Board of Education, 1957–58; Planning Committee, Annual Awards Convocation, U. S. C. School of Education; Group Chairman, Future Teachers' Day; Los Angeles County Fair Committee for Mathematics; Mathematics specialist on Planning and Production Committee, CBS Television series *Learning*, 1957; Reviewing Committee, Report of Commission on Mathematics, C.E.E.B.

Publications: "The Place of Evaluation in the Secondary School Program" (THE MATHEMATICS TEACHER); "Follow-up Data with Implications for Guidance" (*California Journal of Educational Research*); "Notes on the 36th Annual Convention of NCTM," "Challenge of a New School Year" (*California Mathematics Council Bulletin*); "Mathematics Evaluation in a Large City, Los Angeles" (to be published in *Bulletin of the National Association of Secondary School Principals*). Responsible for Los Angeles City Schools publications: "Outline Course of Study for Academic Mathematics," "Instructional Guide for Junior High School Mathematics," "Instructional Guide for High School Mathematics I," "Instructional Guide for High School Mathematics II."

Mildred B. Cole

Mathematics teacher, K. D. Waldo Junior High School, Aurora, Illinois; Instructor, Aurora College: evening classes, "Foundations of Arithmetic" and "Teaching of Arithmetic in the Elementary School."

B.S., University of Illinois; M.S., University of Wisconsin; additional graduate study, University of Colorado.

Elementary teacher, Harvard, Illinois; arithmetic teacher, Grades 5, 6, 7, C. M. Bardwell, Aurora; mathematics teacher, K. D. Waldo Junior High School, Aurora; Laboratory School, University of Wisconsin, Summer 1949.

Member: NCTM; Illinois Council of Teachers of Mathematics; MAA; NEA; Illinois Education Association; AAUW.

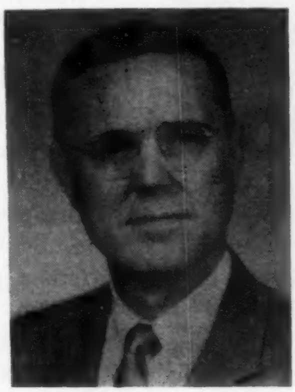
Activities: Member, Mathematics Study Group of Allerton House Conference on Education, 1953–58; ICTM Dele-

gate to Steering Committee of Illinois Curriculum Program, 1955——; Member, Writing Committee of Three for new Illinois Mathematics Curriculum Bulletin, Kdg. 8, "Thinking in the Language of Mathematics"; President, Illinois Council of Teachers of Mathematics, 1953–54; numerous speeches and papers at mathematics conferences and teachers' institutes.

Publications: "Teaching Materials as Keys to Understanding Arithmetic in the Primary Grades"; "Arithmetic Aids for the Middle Grades."

NOMINEES FOR THE BOARD OF DIRECTORS

CAROL V. MCCAMMAN



PHILIP PEAK

CHRISTINE POINDEXTER

OSCAR SCHAAF

HENRY VAN ENGEN

IRENE SAUBLE

Carol V. McCamman

Southeastern Region

Teacher of Mathematics, Calvin Coolidge High School, Washington, D.C.

A.B., M.A., University of California, Berkeley; further graduate work, University of California, University of Chicago, University of Michigan; NSF Institute, University of California, Los Angeles, Summer, 1957.

Examiner in the physical sciences, University of Chicago, 1931–33; research assistant, The Psychological Institute, Washington, D.C., 1933–36; high school mathematics teacher, District of Columbia Public Schools, 1936——; part-time teaching, American University, Catholic University, George Washington University; visiting associate in mathematics, Educational Testing Service, Summer, 1956.

Member: NCTM; MAA; AAAS; NEA;

AAUW; Phi Beta Kappa; Pi Mu Epsilon; Delta Kappa Gamma.

Activities in NCTM: President, District of Columbia Teachers of Mathematics (Affiliated Group, NCTM); Registration Chairman, 1955 NCTM Convention in Washington; Liaison Representative of NCTM on Joint Commission on the Education of Teachers of Science and Mathematics of the AAAS and the AACTE; attendance at conventions and participation in convention programs.

Other activities: Past President, High School Teachers Association, Washington, D.C.; High School Mathematics Contest Committee of D.C.-Maryland-Virginia Section, MAA; Member, department and city textbook and curriculum committees; City-wide Algebra Test Committee; Consultant, Committee on Affiliation, Catholic University; participant, NEA Invitational Conference on the Academically Talented, 1958.

Publications: Article in *Eighteenth Yearbook* of NCTM; assisted in revision of plane geometry text; section on education of the mathematically talented in October, 1958, *NEA Journal;* articles in local educational publications.

Philip Peak

Central Region

Assistant Dean and Associate Professor of Education, University of Indiana, Bloomington, Indiana.

B.A. in Mathematics, Iowa State Teachers College, 1930; M.S. in Mathematics, University of Iowa, 1935; Ph.D. in Mathematics and Education, Indiana University, 1955.

Mathematics and Agriculture teacher, Mechanicsville High School, 1930–35; Head of Mathematics Department, Pierre, South Dakota High School, 1935–38; Assistant Professor Mathematics, Nebraska State Teachers College, Chadron, Nebraska, 1938–42; Head of Mathematics Department of University School and Instructor of Education, Indiana University, 1942–50; Assistant Director of Student Teaching, Indiana University, 1950–56.

Member: NCTM; Central Association of Science and Mathematics Teachers; American Mathematics Association; Sigma Xi; Indiana State Teachers Association; Indiana Academy of Science; Indiana Council of Teachers of Mathematics; NEA; Phi Delta Kappa.

Activities in NCTM: Director of NCTM; Member, Executive Committee; Member, Editorial Committee of THE MATHEMATICS TEACHER; Member *Twenty-second Yearbook* Committee; former member, Film Committee; Local Chairman, Summer meeting, 1955; State Representative for National Council; former member, Board of Directors.

Other activities: Vice President and President of the Central Association of Science and Mathematics Teachers.

Publications: Contributed articles to THE MATHEMATICS TEACHER since 1946; contributed articles to *School Science and Mathematics;* reviewed books for both publications; "Research Before Writing" (*School and Society*, 1948); "Indiana University Chapter, The Society of Sigma Xi" (*American Scientist*, 1956); coauthor, How-To-Pamphlet, "How to Use Film and Film Strips"; coauthor, National Council flyer, "As We See It."

Christine Poindexter

Southwestern Region

Chairman, Department of Mathematics, Central High School, Little Rock, Arkansas.

B.S.E., Arkansas Teachers College; M.A., University of Missouri; additional work, University of Texas; participant in National Science Foundation Institute for High School Mathematics Teachers, Indiana University, 1957.

Teacher of mathematics in high schools of Arkansas for more than 30 years, the past 16 years in Central High School; Chairman of Mathematics Department for ten years; teacher, Little Rock Junior

College and Henderson Teachers College in summer sessions, Little Rock University in night school, 1958.

Member: NCTM; Arkansas Council of Mathematics Teachers; Arkansas Education Association; NEA (Life Member); Delta Kappa Gamma Society.

Activities: President, Arkansas Council of Mathematics Teachers, 1956–58; Consultant in the Teaching of Geometry at University of Arkansas Mathematics Workshops, 1953 and 1954; Sponsor of Mathematics Exhibit at annual convention of Arkansas Education Association for several years; Past President, Little Rock Education Council, 1949–51; Delegate to National Citizenship Conference, Washington, D. C., 1952; Past President of Mathematics Section of A.E.A., before which many talks have been made; Member, Committee on Preparation of the Mathematics Teacher under grant from Ford Foundation.

Oscar F. Schaaf

Western Region

Head, Department of Mathematics, South Eugene High School, and Assistant Professor of Education, University of Oregon, Eugene, Oregon.

B.A., University of Wichita, Wichita, Kansas; M.A., University of Chicago; Ph.D., Ohio State University.

Teacher of Mathematics, Leoti, Kansas, and Anthony, Kansas; Instructor in Education, Ohio State University; Teacher of Mathematics at University School, Ohio State University; Instructor in summer session, Department of Education, Ohio State University; Counselor, Science Teaching Improvement Program, American Association for the Advancement of Science; Assistant Professor in summer sessions, Department of Education, University of Oregon.

Member: NCTM; Oregon Council of Teachers of Mathematics; Ohio Council of Teachers of Mathematics (charter member).

Activities in NCTM: Associate Editor of present *Mathematics Student Journal;* present State Representative for NCTM in Oregon; appearances on convention programs.

Other activities: Secretary and Vice President of Ohio Council of Teachers of Mathematics; Past President of Oregon Council of Teachers of Mathematics; Mathematics Curriculum Consultant for Eugene Public Schools and several other Oregon school systems.

Publications: Consultant and contributor to forthcoming *Handbook for Oregon Secondary School Mathematics Teachers;* Advisor and Editor of revision of the *Oregon Mathematics Scope and Sequence;* many articles in professional periodicals.

Henry Van Engen

North Central Region

Professor of Education and Mathematics, University of Wisconsin, Madison, Wisconsin.

A.B., Nebraska Wesleyan University; Ph.D., University of Michigan.

Elementary, Junior High School, and Senior High Schools, Nebraska, Michigan, and Ohio; Western Reserve University; Kansas State University; Iowa State Teachers College.

Member: NCTM; MAA; AMS; Sigma Xi; Phi Kappa Phi; Phi Beta Kappa; Pi Mu Epsilon; Kappa Mu Epsilon.

Activities in NCTM: Editor, THE MATHEMATICS TEACHER; Member, Board of NCTM; Member, Elementary Curriculum Committee.

Other activities: Past President, Kappa Mu Epsilon; Member, SMSG Advisory Committee, Commission on Mathematics.

Publications: Numerous articles appearing in THE MATHEMATICS TEACHER, *School and Society*, *School Science and Mathematics*, *The Arithmetic Teacher*, and other magazines; chapters in various yearbooks; coauthor of textbooks.

Irene Sauble

North Central Region

Director of Exact Sciences, Detroit Public Schools, Detroit, Michigan.

A.B., University of Michigan; M.A., University of California.

Teacher of Mathematics, Northwestern High School, Detroit, Michigan; Supervisor of Mathematics and Director of Exact Sciences, Detroit Public Schools; Part-time Associate Professor of Teaching of Mathematics, Wayne State University, Detroit 1936–51; Instructor during summer sessions at: Claremont Colleges, Claremont, California, 1931; Loyola University, Chicago, Illinois, 1932; Chico State College, Chico, California, 1933; Served on staff of workshops in Oakland, California 1950–51.

Member: NCTM, NEA, Michigan Council of Teachers of Mathematics; Detroit Council of Teachers of Mathematics, Michigan Education Association; Phi Beta Kappa.

Activities in NCTM: Vice-president—Elementary School Level, 1952–54; Program chairman for 1953 summer meeting of NCTM; Member of the Elementary Curriculum Committee, 1955–58; North Central Regional Representative for Affiliated Groups, 1954–56; Member of Twenty-fifth Yearbook Committee, 1956–59.

Publications: author of: "The Enrichment of the Arithmetic Course—Utilizing Supplementary Materials and Devices," Sixteenth Yearbook, National Council of Teachers of Mathematics, 1941; "Teaching Fractions, Decimals and Per Cent: Practical Applications," Arithmetic 1947, University of Chicago Monograph, Number 63; "Applications of the Film in Mathematics," The Film and Education, Philosophical Association of America, N. Y., 1948; "Development of Ability to Estimate and Compute Mentally," *The Arithmetic Teacher*, April 1955.

Co-author of: "The Measurement of Understanding of Elementary School Mathematics," Forty-fifth Yearbook, Part I, National Society for the Study of Education, 1941; "Reading in the Content Fields," *The Reading Teacher*, 1956; "Arithmetic We Need, Grades 3–8 (with Buswell and Brownell) Ginn and Company, 1955.

Experimental Program at Illinois

Grade school pupils are being guided to discover mathematics principles for themselves in a new arithmetic project at the University of Illinois which will be carried on over a five-year period under sponsorship of the Carnegie Corporation of New York.

Grants totalling well over half a million dollars to be used in connection with this project and to continue the University's high school mathematics project, now in its eighth year, were announced by the Carnegie Corporation trustees today, the arithmetic project receiving $307,400 and the high school project $282,600.

Experimental grade school classes in arithmetic are being carried on to determine what mathematics can be learned—or discovered—by these children. Information thus gained will be used to formulate a new grade school mathematics program including geometry, algebra and other topics new to the elementary school. The new materials will be adaptable for use—in whole or in part—by any reasonably able teacher following summer study or intensive on-the-job self-training.

Director of the arithmetic project is Prof. David Page. Experimental classes are now being conducted in the fifth grade, Westview School, Champaign; fourth grade, Leal School, Urbana; and with younger children in a private school near Philadelphia, Pa. Eventually the project will involve all eight grades.